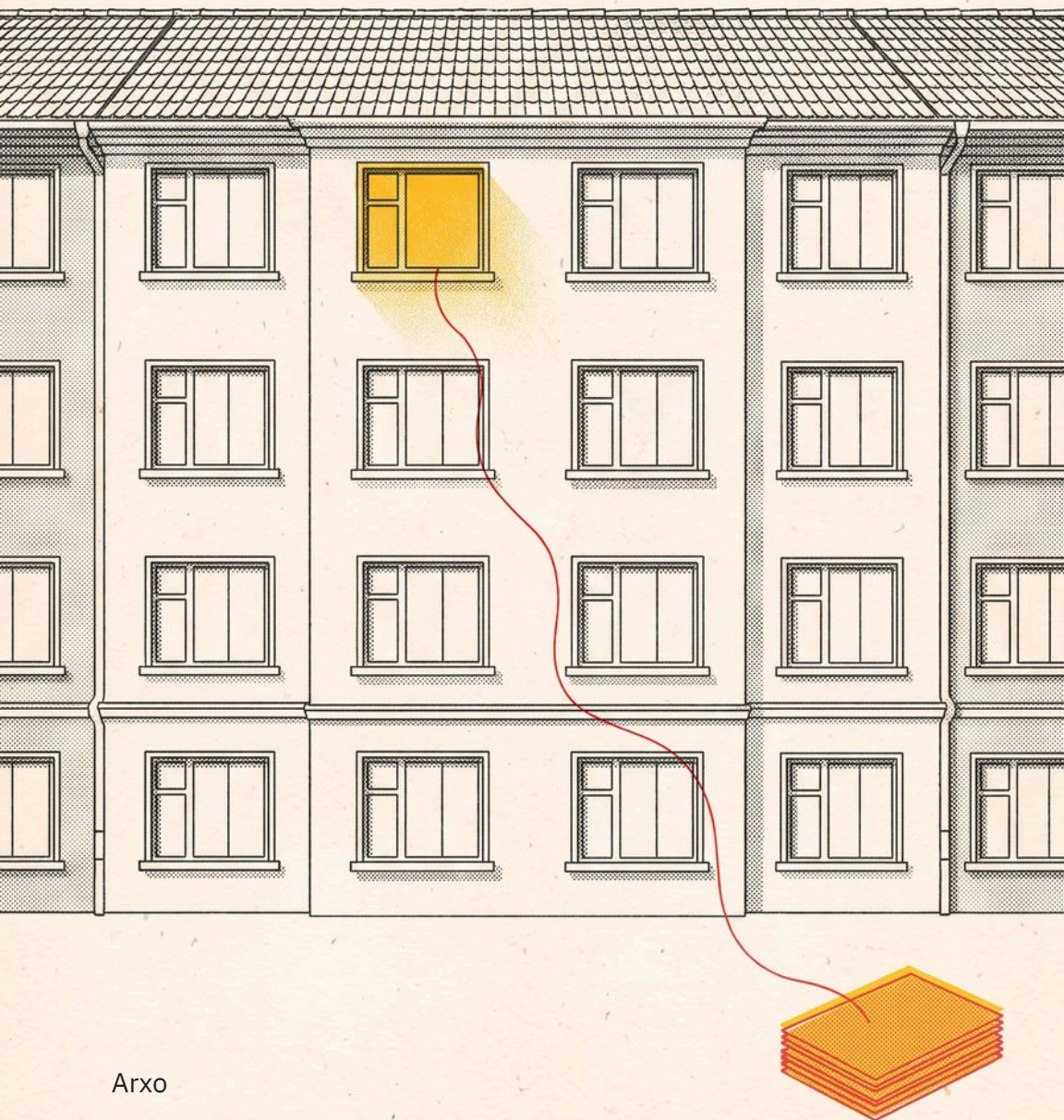


Law as Code in Practice: German Housing Benefit (Wohngeld)

Vom Bundesgesetzblatt zum Bescheid — One Family in Weimar



LAW AS CODE BY COUNTRY · VOLUME 2

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Law as Code by Country · Volume 2

First of all a demonstration of what modelling legislation with Arxo can do: an independent model built from the published texts of the WoGG, the WoGV and the EStG; it does not claim legal accuracy or completeness, it is not an implementation by or for the BMWWSB or any Wohngeldbehörde, and its answers are not legal advice. The Vogts and the other households in this book are fictional; the Vogts' facts and numbers are those of the ministry's worked example.

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Preface: Why Germany, and Why One Family

This book is first of all a demonstration of what modelling legislation with Arxo can do. Its model is an independent reading of the published German texts, built for teaching; it does not claim legal accuracy or completeness, it is not an implementation by or for the BMWSB or any Wohngeldbehörde, and its answers are not legal advice. For a real claim, rely on the official texts and on the Wohngeldbehörde.

Germany is a good place to learn law as code. Federal law is free to read on gesetze-im-internet.de, and German statutes are free to reuse (§ 5 UrhG). Its civil code gave the world a general part and a systematic order that many codes still follow, and its *Rechtsinformatik* — legal informatics — has been a field since 1970. And for the benefit this book follows, the federal ministry responsible for housing, the BMWSB, publishes worked examples of its own formula: the incomes, rents and amounts they print are an official check on the model's numbers. GETTSIM models German taxes and transfers in the open, and Catala, from Inria, showed that a statute can be written as code beside its own text. This book stands on that work, and its acknowledgements say how.

The book follows one benefit, *Wohngeld* — Germany's housing benefit — and one family: the Vogts, a couple with two children in a rented flat in Weimar. The names are ours; their facts and numbers are those of the ministry's *Rechenbeispiel 7* for 1 January 2025. Other households join where a chapter needs them: a woman whose job centre already pays her housing costs, a neighbour who owns her flat, a single man in Ludwigshafen am Rhein for the final exercise. Every answer in the book comes from a run of an executable model of the published rules,

and every answer can be traced to the section, the edition and the facts behind it.

That model is an independent Arxo model built from the published texts of the WoGG, the WoGV and the EStG. It is not an implementation by or for the BMWBSB or any Wohngeldbehörde, and nothing in this book describes how a particular office processes a file.

What the book is for

The model answers more than “how much”. A housing-benefit file raises questions a calculator does not: which edition of the annexes applies on the date of the question, what happens on the day the values change, what the answer is when a fact is missing or two records disagree, which questions the Act leaves to the authority, which facts were decided by another authority, and how a saved answer is checked again a year later. Each chapter takes one of these questions and shows what the rules establish, what remains open, and who must act next.

The book is written for the people who build and run such systems together: lawyers and staff of housing-benefit authorities, programme analysts, and developers. You do not need to know Arxo, and German helps but is not required: every German term is glossed in English where it first appears, the sections are quoted in German with an English gloss, and a glossary at the end lists the terms.

What the book is not

This is not legal advice, and it is not a guide to anyone’s entitlement. The households are fictional. The amounts are answers to dated questions about stated facts: a different date, a different fact, or a later edition of a text can give a different answer, and the book shows exactly how. The ministry’s worked examples are examples; as its own brochure says, they establish no legal claim.

A short book

This volume is shorter than the first in the series: one benefit, one family, ten chapters and a final exercise — what a reader needs to follow a decision from the gazette to the claim. The procedure after the amount

— the Bescheid, duties to cooperate and deadlines — and the bill for 2027 are planned for a later edition.

Dates

Every question in this book is asked at a legal date — for the Vogts, 1 January 2025, and 31 December 2024 when the chapter compares the two editions — against the texts and annex values in force on that day. Where a chapter shows a recorded run, it gives the date of the recording as well: most answers were recorded on 12 October 2026. The two are different: a question about 1 January 2025 can be asked again years later and must give the same answer from the same texts.

An online edition

The same text is also an online course. The book and the course are one text in two forms; corrections reach both.

How to Read This Book

Chapters are numbered from 0, like the lessons of the online course, so a reference to “Chapter 6” means the same thing in both. Each chapter opens with a question from the family and asks you to predict the answer. It then quotes the German text that answers it with an English gloss, shows the part of the model that carries it and the model’s answer, changes one fact to see what moves, and ends with what the answer gives the authority and an exercise. Solutions are at the end of the book: try the exercise first.

Three ways through

Read the book in order if you can; it is short. If you come with one job in mind, these paths keep the thread:

- **Lawyers and staff of housing-benefit authorities** — Chapters 0, 1, 3, 6, 7 and 8: which section decides each answer, which text applies on which day, how one decision reads step by step, where the Act leaves the decision to the Wohngeldbehörde, when another authority’s decision excludes a person, and what the model says when facts are missing.
- **Programme analysts** — Chapters 0, 1, 2, 3 and 4: which fact changes an answer and by how much, how the household, the rent and the income enter the formula, and how the biennial update moves a running grant.
- **Developers** — Chapters 0, 3, 4, 5 and 9: how a case holds facts and questions, how the formula appears in the proof, how a scenario is written against the ministry’s examples, and how a saved answer is replayed.

Every chapter opens with a short note naming what each of these readers should take from it.

Reading an answer

The model's answers are more precise than “yes” and “no”. Every answer to a question carries one of four truth statuses — the same names in every volume of the series:

- **Established** — the facts and rules support the statement and do not support its opposite. The answer names the rule and the section behind it.
- **Refuted** — they support the opposite and do not support the statement. Not met in this book.
- **Not established** — they support neither. This is not zero and not a refusal: often a fact is missing, and the model names the open premise.
- **Contradiction** — they support both, as when two records of the file disagree on a fact. The answer keeps both, with their sources, for review instead of choosing silently.

Beside the truth status, an answer can say why there is no number, or no final answer yet:

- **Requires judgment** — the Act leaves a question to the authority, such as whether claiming would be abusive. The amount can be known while the claim waits; the answer names the question and the Wohngeldbehörde it is addressed to.
- **Outside the model** — the case is one the model does not cover, such as an owner-occupier's Lastenzuschuss. A flag names the reason; it says nothing about the person's entitlement.
- **Past the model's values** — the date of the question lies after the last annex values this model holds. The answer names the missing values instead of carrying the old ones forward; it does not say that no values are in force.

Chapter 0 sorts the answers a housing-benefit office gives into five kinds; Chapter 8 adds records that disagree and dates past the model's values.

Amount and claim

The book keeps two questions apart: how much the formula of § 19 gives, and whether a claim to that amount exists under §§ 19–21. The first can be established while the second is not — Chapter 6 is about exactly that.

Commands

Most chapters end with the commands behind their answers: a question asked of the case package with `law ask`, or a scenario file of the canon run with `law test`. The appendix on reproduction explains how to run them and how a saved answer is replayed: `law eval` evaluates the saved answer again — without the source files or a rebuild — and must give the same bytes.

PART I

The Task



What a Wohngeldbehörde Decides, and Why It Is a Law-as-Code Problem

By the end of this chapter you will know what one family in Weimar receives, which sections decide it, and why “how much” and “is it owed” are two different questions.

Lawyer: which § decides each answer.

Analyst: which fact flips the answer.

Developer: which command reproduces it.

1. The benefit and the authority

Wohngeld is Germany’s housing benefit: a monthly subsidy towards housing costs for households whose income is too high for basic income support but too low to carry the rent alone. The federal Housing Benefit Act — the *Wohngeldgesetz*, WoGG — sets the conditions and the formula. A federal ordinance, the *Wohngeldverordnung* (WoGV), sorts every municipality into a rent level. The amounts themselves are fixed in three annexes to the Act, the *Anlagen*, which the federal government updates every second year.

The decision is made locally. A *Wohngeldbehörde* — the housing-benefit office of a district or city — receives the application, establishes the facts and issues the decision notice, the *Bescheid*. The federal ministry

responsible for housing, the BMWSB, keeps the Act and its annexes up to date and publishes worked examples of the calculation.

There are two kinds of Wohngeld. A *Mietzuschuss* helps tenants; a *Lastenzuschuss* helps owner-occupiers with the costs of their own home. This book follows tenants.

2. One family, one question

Meet the Vogts — a fictional family whose facts are those of the ministry’s official example: *Rechenbeispiel 7* in the BMWSB brochure *Beispiele für die Berechnung des Wohngelds*, as of 1 January 2025. The names are ours; the numbers are the ministry’s.

Family card	
Household	Katrin and Markus Vogt, married; Lena (10) and Paul (6)
Home	a rented flat in Weimar, Thuringia
Rent	€580 a month, without heating (<i>Bruttokaltmiete</i>)
Income	Katrin earns €2,240 gross a month; nobody else in the household has income
Date of the question	1 January 2025
Still open	nothing — the authority has assessed everything it must assess

The ministry’s example gives the children no ages; the case records 10 and 6, which do not affect the amount.

Their question: *how much Wohngeld do we receive from January 2025, and on which basis?*

Before you read on, make a prediction. The ministry’s example was published for 1 January 2025. Would the same family, with the same rent and income, have received more, less or the same on 31 December 2024?

Question	Model's answer (recorded 12 October 2026)	Section
Monthly assessed income	€1,710.00 — established	§ 13 Abs. 2, §§ 14–16 WoGG
Rent taken into account	€777.80 — established	§ 11 Abs. 1, § 12 Abs. 1, 6, 7 WoGG
Monthly Wohngeld by the formula	€485 — established	§ 19 WoGG with Anlagen 2 and 3
Claim to €485	established	§§ 19–21 WoGG
The same family on 31 December 2024	€463 — established	§ 44 Abs. 1, § 43 WoGG

The ministry’s 2025 example gives the first three numbers — €1,710.00, €777.80 and €485 — and the model matches them to the euro. The last line applies the values in force on 31 December 2024 to the same facts and gives €463; it answers your prediction: the family would have received €22 less. Nothing about the family changed; the law did, overnight, and the Act itself says which text to use on which day. Chapter 1 is about exactly that.

3. Five kinds of answer

A housing-benefit office does not give one kind of answer. The model keeps five apart, because confusing them is how people are wronged:

Answer	When	What the model returns
An amount	every premise is established	the amount, <i>established</i> , with the path through the sections
No claim by law	an excluding fact is established — another benefit that covers housing costs, an amount under €10	a ground of exclusion, <i>established</i>
Not enough facts	the rent level, the rent, a member's status or an income is missing	<i>not established</i> — no amount, and not zero
The authority must assess	the Act leaves a judgment to the office: whether claiming would be abusive (§ 21 Nr. 3), whether two people form a community of responsibility “nach verständiger Würdigung” (§ 5 Abs. 1 S. 2 Nr. 3)	a request addressed to the Wohngeldbehörde
Outside the model	a Lastenzuschuss, or a kind of income the model does not cover	a flag naming why there is no number

The third and fifth lines matter most to a family. “We do not have your rent yet” is not “you get nothing”, and “our model does not cover owner-occupiers” is not “the law gives you nothing”. Chapters 6–8 take these answers one by one.

4. What an answer contains

Every answer from the model carries four things:

1. **the facts used** — the family's data, each with its origin: entered by the household, or decided by an authority;
2. **the parameters applied** — rent ceilings, heating relief, the formula's values — each tied to the row of the Anlage that was in force on the date, and to the issue of the *Bundesgesetzblatt* (BGBl), the federal law gazette, that published it;
3. **the operations** — a proof graph, rule by rule, from the facts to the result;
4. **the section behind each rule**, in the edition in force on the date of the question.

German administrative law asks for the same things in its own words. A written decision must state its essential factual and legal grounds (§ 35 Abs. 1 SGB X). A decision may be issued fully automatically only where

there is no reason to have an official handle the individual case (§ 31a SGB X). The model shows what an answer rests on; it does not by itself make a decision lawful.

5. Why this is a law-as-code problem

The Vogts' €485 draws on the WoGG, the WoGV with its list of rent levels, the Income Tax Act (EStG) for the definition of income, two amending ordinances and an amending Act. On top of that:

- **the text changes on a fixed date** — every second year, the annexes are replaced, and § 44 says the new values apply from that day;
- **the model's values run out** — the model holds annex values for dates up to 31 December 2026; the next ones are not yet enacted, and the model must not invent them;
- **some conditions need the office** — the Act leaves “abusive” and “nach verständiger Würdigung” to the authority;
- **some facts come from other authorities** — whether a member receives basic income support with housing costs is decided by a job centre, not by the formula;
- **one amount, many readers** — the household, the caseworker, the auditor and the court must all be able to follow it.

6. Standing on German shoulders

This book builds on German work that is older than the phrase “law as code” — the civil code's systematic order, *Rechtsinformatik* since 1970, free federal law online, the ministry's own worked examples, GETTSIM and, beyond Germany, Catala. The acknowledgements give each of these its due.

7. What this book adds

New to law as code? Skim this table now and come back to it after Chapter 3.

In Arxo	Chapter
The date of the question selects the edition and the row of each Anlage, and the answer names both	1, 9
Past the last values this model holds, the answer is a refusal with a reason, not zero	1, 8
A missing premise the model checks is <i>not established</i> ; it is not treated as “no”	2, 8
A question the Act leaves to the authority becomes a request addressed to the Wohngeldbehörde, while the amount is already known	6
An exclusion rests on another authority’s decision entered as a fact, with its origin	7
Two records that disagree give a <i>contradiction</i> status, kept for review	8
A saved answer can be replayed from its fingerprints, byte for byte	9

Catala’s strengths come first wherever the book compares: exact decimals, money and dates, exceptions written beside the text, tests and execution traces. Appendix F names the version and the date of the check, once for all comparisons.

8. A few German terms

German	English	Meaning in this book
<i>Wohngeld</i>	housing benefit	the monthly subsidy this book follows
<i>Mietzuschuss / Lastenzuschuss</i>	rent subsidy / owner-occupier subsidy	the tenant’s and the owner’s kind of Wohngeld; the second is outside the model
<i>Wohngeldbehörde</i>	housing-benefit authority	the local office that decides
<i>Bescheid</i>	decision notice	the written decision the household receives
<i>Haushaltsmitglied</i>	household member	a person whose income and needs count
<i>Bruttokaltmiete</i>	gross rent without heating	the rent including service charges, excluding heating and hot water

German	English	Meaning in this book
<i>zu berücksichtigende Miete</i>	rent taken into account	the rent the formula uses, after ceilings and heating relief
<i>Mietenstufe</i>	rent level I–VII	the level of rents in a municipality, set by the WoGV
<i>Gesamteinkommen</i>	assessed household income	income as the WoGG defines it, not as a payslip shows it
<i>Fortschreibung</i>	biennial update	new values in the annexes every second year
<i>Anlage</i>	annex	the tables of amounts attached to the Act
<i>Bundesgesetzblatt (BGBl)</i>	federal law gazette	where Acts and ordinances are published

9. Explore the case

All answers in this chapter come from one case package, `familie-weimar`, which holds four files over the same model. The commands below are patterns: `<case package>` stands for the path of that package in a checkout of the Arxo repository, given in full in the appendix on reproducing the examples.

```
law ask <case package> --case Familie2025 --query-json \
  queries/wohngeld-485.json
law ask <case package> --case Familie20241231 \
  --query-json queries/wohngeld-463.json
```

The first asks whether the monthly Wohngeld on 1 January 2025 is €485; the second asks the same family’s question on 31 December 2024 with €463. Chapter 3 shows what comes back. These answers were recorded on 12 October 2026 with `law 0.1.12`.

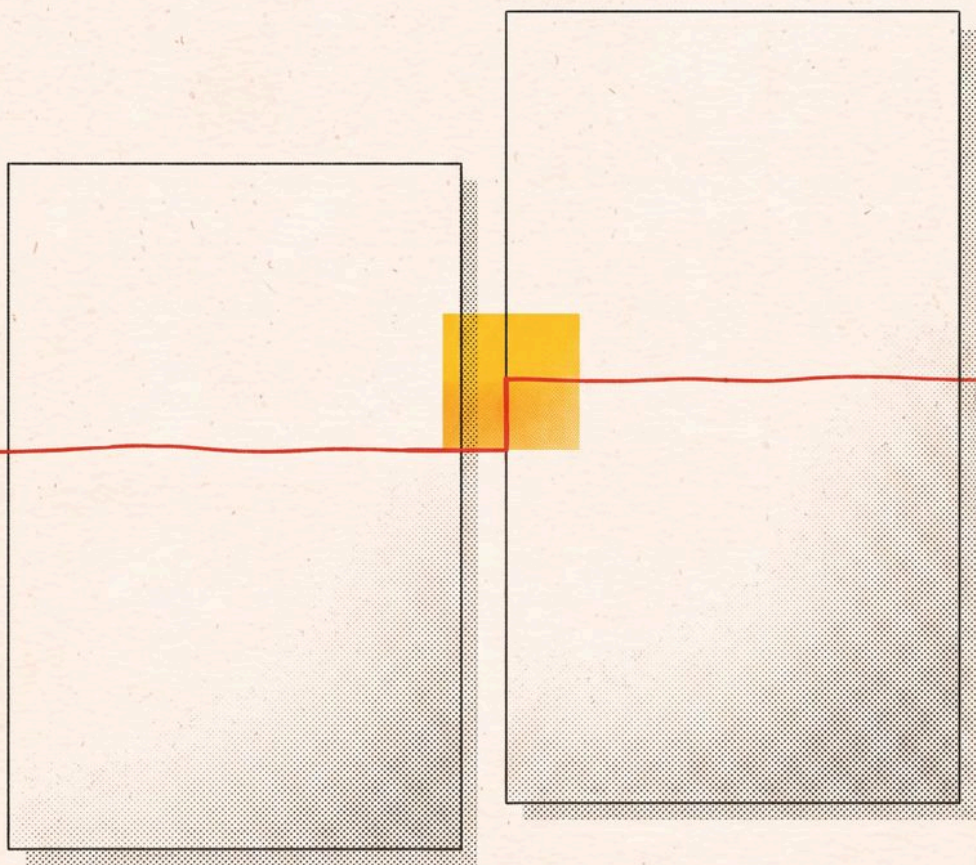
Exercise

Use the table in section 2 and the five kinds of answer in section 3.

1. The office has the Vogts' income but not yet their rent. Which kind of answer does the model give for the monthly Wohngeld, and why is it not €0?
2. Which amount in section 2 changes between 31 December 2024 and 1 January 2025, and which calculated inputs stay the same?
3. A colleague says: "The formula gives €485, so the family has a claim to €485." What else must hold?

PART II

How the Model Is Built



Which Text Applies? The Bundesgesetzblatt and the § 44 Transition

By the end of this chapter you will be able to say which edition of the Act and which row of each annex an answer used, why one family gets two amounts on two neighbouring days, and what the model does past the last values it holds.

Lawyer: which provision orders the change of values, and from when.

Analyst: which dates give which amounts.

Developer: where the answer records the edition and the row it used.

1. The question

The Vogts from Chapter 0 ask the same question twice: once for 31 December 2024 and once for 1 January 2025. Their rent, their income and their household are identical on both days.

Predict before you read on: should a model give the same amount on both days, or two different amounts? And if two, which text tells it to switch?

2. A change the Act itself orders

The tables of the formula are not written in the sections of the Act but in its annexes: Anlage 1 holds the rent ceilings, Anlage 2 the values a, b and c of the formula, Anlage 3 the minimum values and the order of the calculation steps. § 43 Abs. 1 WoGG orders that these values be updated

“zum 1. Januar jedes zweiten Jahres” — on 1 January of every second year — by an ordinance, the *Wohngeldfortschreibungsverordnung*, WoGFV. A few amounts stand in the sections themselves: the heating relief and the climate component in § 12, the allowances in § 17.

What happens to a household whose decision covers the date of an update? § 44 Abs. 1 WoGG answers:

Ist Wohngeld vor dem Inkrafttreten der Fortschreibung des Wohngeldes (§ 43) bewilligt worden und dauert mindestens ein Teil des Bewilligungszeitraums nach dem Inkrafttreten der Fortschreibung noch an, so ist abweichend von § 41 Absatz 2 von Amts wegen über die Leistung des Wohngeldes für den Zeitraum vom Inkrafttreten der Fortschreibung bis zum Ende des bisherigen Bewilligungszeitraums neu zu entscheiden. Bei der Entscheidung sind die Berechnungsgrößen des Wohngeldes nach § 43 Absatz 1 Satz 1 Nummer 1 bis 6 in der ab dem Inkrafttreten der aktuellen Fortschreibung geltenden Fassung anzuwenden.

In English: if Wohngeld was granted before an update and the period of the grant runs past the update’s entry into force, the authority decides again, of its own motion, for the time from that day; and it applies the values of the new update. There is no “at least the old amount” clause. So the same family can lawfully receive two different amounts on two neighbouring days.

3. Two editions in the model

The model holds the values of each update with the dates they cover, and the publication each came from:

Values	Dates in the model	Published in
Anlagen 2 and 3 of the <i>Wohngeld-Plus-Gesetz</i> ; Anlage 1 of the first WoGFV	1 January 2023 – 31 December 2024	BGBl. I 2022 S. 2160 (Nr. 48); BGBl. I 2021 S. 1369 (Nr. 28)
Anlagen 1–3 of the second WoGFV	1 January 2025 – 31 December 2026	BGBl. 2024 I Nr. 314

Each publication is pinned by the fingerprint of its bytes. The 2024 issue was taken from the official portal recht.bund.de; the earlier issues from

the published copies on OffeneGesetze.de. The model records which is which, so a reader can check the source of every row.

For the Vogts, three values change on 1 January 2025:

Value	31 December 2024	1 January 2025
Rent ceiling, 4 members, Mietenstufe III (Anlage 1)	€736	€766
b, 4 members (Anlage 2)	2.251×10^{-4}	2.163×10^{-4}
c, 4 members (Anlage 2)	2.000×10^{-5}	1.760×10^{-5}

The heating relief (€197.80 for four members) and the climate component (€34.40) did not change. Neither did the family’s rent taken into account: their rent of €580 stays under the ceiling either way, so M is €777.80 on both days. What moves is the formula.

4. The answers

Case	Date	Question	Model’s answer (recorded 12 October 2026)
Familie20241231	31 December 2024	Is the monthly Wohngeld €463?	established
Familie20241231	31 December 2024	Is it €485?	not established
Familie2025	1 January 2025	Is it €485?	established
Familie2025	1 January 2025	Is it €463?	not established

That settles the prediction: two amounts, €463 and €485, and § 44 Abs. 1 is the text that orders the switch.

The answer does not just state the amount. Each step that uses an annex records which row it took. For the Vogts’ formula on 1 January 2025 the proof names:

```
WertMitglieder4#2025-01-01
WertBMitglieder4#2025-01-01
WertCMitglieder4#2025-01-01
HoechstbetragMitglieder4MietenstufeIII#2025-01-01
```

and each row points to its anchor, the text of Anlagen 1–3 in the second WoGFV, BGBl. 2024 I Nr. 314. On 31 December 2024 the same steps name the rows #2023-01-01: the values a, b and c point to Anlage 2 of the *Wohngeld-Plus-Gesetz*, the ceiling to Anlage 1 of the first WoGFV. Nobody has to remember which table was in force: the answer says so.

5. Where the model’s values end

The model holds parameter values for dates up to 31 December 2026: the values of the second WoGFV, which § 43 Abs. 1 meant to last until the next biennial update on 1 January 2027. Which values apply from that day depends on what is enacted and published; until the model holds such a text, it has no values for 2027 — and it says so rather than reusing the old ones or returning zero. A government bill that would change the rules for 2027, BT-Drs. 21/8284, is before the Bundestag; until it is enacted and published, the model holds no 2027 values.

The scenarios of the edition suite show all four edges, for the same family:

Date	Model’s answer
31 December 2024	€463, established
1 January 2025	€485, established
31 December 2026 — last day the model holds values for	€485, established
1 January 2027	no amount: MISSING_PARAMETER_VALUE — the model holds no value of the annexes for this date
before 1 January 2023	no amount: EDITION_NOT_APPLICABLE — the model holds no earlier edition

For a housing-benefit office, the last two lines are the important ones. A system that quietly carries 2026 values into 2027 produces numbers that look right without saying which text they rest on. A refusal with a reason tells the office exactly what is missing: the values for the new period, with their source.

6. A second household: when the rent ceiling matters

The ministry's *Rechenbeispiel 6* is a different family of four in Munich, rent level VII, with a rent of €1,225 and an assessed income Y of €2,227.25 a month. Here the ceiling matters. On 31 December 2024 the ceiling plus the climate component is €1,129.40, so the rent taken into account is $€1,129.40 + €197.80 = €1,327.20$, and the Wohngeld is €621. On 1 January 2025 the ceiling rises to $€1,139 + €34.40 = €1,173.40$, the rent taken into account becomes $€1,371.20$, and the Wohngeld €691. Here the update moved two things at once: the ceiling and the formula.

7. What this gives the authority

When the values change on 1 January, a Wohngeldbehörde has to decide again for every running grant. With the edition and the row recorded in each answer, a caseworker can see which values a decision used, a reviewer can check them against the gazette, and a court can follow the change line by line. When the model's values run out, it stops the line instead of extending it.

8. Where Catala stands on this

Catala keeps legal text beside executable code and supports dates (*Catala book, introduction*¹ and *ch. 5.2*²). Arxo provides built-in declarations for source anchors and pinned publications, and records in each answer the row of the annex it used. The Catala 1.3.0 book does not describe that publication-binding form. A fingerprint identifies saved bytes; legal interpretation requires a separate review. The authority can keep the publication records with the dated file and tell later source changes apart without relying on recollection.

¹<https://book.catala-lang.org/en/>

²<https://book.catala-lang.org/en/5-2-types.html>

9. Run it

```
law ask <case package> --case Familie20241231 \
  --query-json queries/wohngeld-463.json --out \
  runs/2024-12-31
law ask <case package> --case Familie2025 --query-json \
  queries/wohngeld-485.json --out runs/2025-01-01
```

In each saved `result.json`, the proof node of the formula carries the attribute `a3Selections`: the rows of Anlage 2 the step used. The edges of the values the model holds are scenarios of the canon's edition suite (`law test <canon> --file tests/redaktionen.lawtest: 6 of 6 passed on 12 October 2026 with law 0.1.12`).

Exercise

1. A grant for the Vogts runs from 1 October 2024 to 30 September 2025. Under § 44 Abs. 1, what must the Wohngeldbehörde do in January 2025, and which amounts apply before and after?
2. A colleague proposes: “If the 2027 values are not published yet, just use the 2026 values until they are.” What does the model return instead, and why is that safer for the office?
3. In the Munich household, why does the rent taken into account change on 1 January 2025, while for the Vogts in Weimar it does not?

Household, Rent and Income

By the end of this chapter you will be able to follow the three inputs of the Wohngeld formula — the number of household members, the rent M and the income Y — from the family's facts to the sections that turn them into numbers.

Lawyer: which sections define household, rent and income.

Analyst: which deduction moves Y and which component moves M.

Developer: which facts the case must supply, and what happens when one is missing.

1. The question

The Vogts' payslip says €2,240 gross a month. Their tenancy agreement says €580. The formula of § 19 WoGG uses neither number as it stands: it uses an income Y and a rent M as the Act defines them.

Predict before you read on: is the Vogts' Y higher or lower than €2,240 — and is their M higher or lower than €580?

2. Who belongs to the household

Wohngeld is computed for a household, and the formula's values a, b and c depend on how many members are counted. § 5 Abs. 1 WoGG defines a household member: the person entitled to claim, if the home is the centre of their life, and everyone who shares it with them as a spouse not permanently separated, a partner, a relative, a foster child or parent — or someone who lives with a member so closely that, “nach verständiger

Würdigung”, a mutual will to take responsibility for each other must be assumed.

For the Vogts the case enters, for each of the four people:

- that the flat is the centre of their life (*Lebensmittelpunkt*);
- their relationship to Katrin, the applicant — Markus as her spouse not permanently separated, the children as relatives;
- their residence status;
- that none of them receives a benefit that would exclude them under § 7 (Chapter 7);
- that their income details are complete.

From these facts the model derives four household members, all counted. Each fact matters: section 6 removes one and shows the household change.

The phrase “nach verständiger Würdigung” — on a reasonable assessment — is the Act leaving a judgment to the authority. The model does not guess it: when a case needs that judgment, the answer asks the Wohn-geldbehörde (Chapter 6). The Vogts are married, so the question does not arise.

3. The income: from €2,240 to €1,710

The Act counts income in four steps, each in its own section:

Step	Section	The Vogts
Annual income under the EStG: positive <i>Einkünfte</i>	§ 14 Abs. 1 WoGG with § 2 Abs. 1, 2 EStG	€26,880 wages (12 × €2,240)
minus the employee’s flat-rate deduction for work expenses, unless higher expenses are proven	§ 9a Satz 1 Nr. 1 Buchst. a EStG	– €1,230 → €25,650
minus 10 % for each of: income tax, health and care insurance contributions, pension contributions — where these are expected to be paid	§ 16 WoGG	Katrin pays health/care and pension contributions, no income tax is expected: – 20 % → €20,520
minus allowances and maintenance deductions (none here); one twelfth	§§ 13, 17, 18 WoGG	€20,520 ÷ 12 = €1,710.00

§ 16 is worth reading in German, because it is the step people get wrong:

Bei der Ermittlung des Jahreseinkommens sind von dem Betrag, der sich nach den §§ 14 und 15 ergibt, jeweils 10 Prozent abzuziehen, wenn zu erwarten ist, dass im Bewilligungszeitraum die folgenden Steuern und Pflichtbeiträge zu leisten sind: 1. Steuern vom Einkommen, 2. Pflichtbeiträge zur gesetzlichen Kranken- und Pflegeversicherung, 3. Pflichtbeiträge zur gesetzlichen Rentenversicherung.

In English: from the annual income, deduct 10 % for *each* of the three — income tax, statutory health and care insurance, statutory pension insurance — that is expected to be paid during the grant period. Not the actual tax and contributions: flat rates of 10 %. The case records, for each member, which of the three apply; for Katrin, the second and third. The ministry's example writes the same calculation in one line: (€2,240 – €102.50) × 0.8 = €1,710.00, where €102.50 is the monthly share of the €1,230 flat rate. The model reaches €1,710.00 in the steps above and **establishes** it.

4. The rent: from €580 to €777.80

The rent the formula uses is built in three steps.

The rent under § 9. The agreed payment for the use of the home, including service charges, minus what the Act leaves aside — above all the costs of heating and hot water (§ 9 Abs. 2 WoGG with § 6 WoGV). The Vogts' €580 is already a *Bruttokaltmiete*: service charges in, heating out. The case declares that the rent details are complete, so nothing further is deducted: the rent under § 9 is €580.

The ceiling. § 11 Abs. 1 WoGG takes the rent only up to a ceiling: the *Höchstbetrag* of Anlage 1 for the number of members and the rent level, plus a *Klimakomponente* — a climate component — from § 12 Abs. 7. The rent level comes from the place, not from an input: the WoGV's list puts the town of Weimar at **Mietenstufe III**. For four members at level III in 2025, the ceiling is €766, the climate component €34.40: together **€800.40**. The Vogts' €580 is below it, so all of it counts.

The heating relief. § 12 Abs. 6 adds a fixed amount for heating, whatever the actual heating bill: for four members, €25.80 for the carbon price and €172 as a permanent heating component, **€197.80** in all.

So $M = €580 + €197.80 = \text{€}777.80$, established. That answers the second half of the prediction: M is higher than the rent, because the Act adds heating relief that the agreed rent does not contain.

5. Two answers to the prediction

Input	On the paper	In the formula
Income	€2,240 a month gross	$Y = \text{€}1,710.00$
Rent	€580 a month	$M = \text{€}777.80$
Household	four people	4 members, all counted

Y is lower than the payslip — by the flat-rate expenses and two 10 % deductions. M is higher than the rent — by the heating relief.

6. Change one thing

Income tax becomes due. Suppose Katrin's income tax were also expected in the grant period. § 16 would then deduct 10 % three times instead of twice: $€25,650 \times 0.7 = €17,955$ a year, €1,496.25 a month. Y falls by €213.75 to €1,496.25, and the Wohngeld rises from €485 to **€542** — both values from a run of the model.

Markus's life centre is not established. Remove the fact that the flat is the centre of Markus's life — not a finding that he lives elsewhere, just a fact the file no longer contains. § 5 Abs. 1 makes that fact a condition of membership, so the model no longer counts him — and the case changes in three places at once. The household has three members, so Anlage 2 gives other values of a , b and c , and the rent taken into account uses the components for three: $€580 + €170.20 = \text{€}750.20$. With Markus not counted, the household the model sees is Katrin living alone with her children, so § 17 Nr. 3 gives the single-parent allowance of €1,320 a year: $Y = (€20,520 - €1,320) \div 12 = \text{€}1,600.00$. The model then computes **€351** — the answer for a mother living alone with two children, which is not the Vogts' situation if Markus does live there. One fact, and the model follows every section it touches; and it does not flag that a fact about Markus is missing. So €351 is the result of an incomplete file, not an answer to give the family: until the household's composition is established, it must not be sent to them. Whether Markus lives in the flat

is the caseworker's question to settle, and the answer moves the amount by €134. (The single-parent allowance also has conditions of its own in § 17 Nr. 3; the case's facts about the children meet them.)

7. What this gives the authority

A caseworker sees the household, the rent and the income as the Act defines them, each with its section and each with the facts it rests on. A missing premise the model checks — the place, the rent, a member's status under § 7, an income detail — appears as a missing fact, not as a silent zero or a silent default; a fact that decides who belongs to the household must be settled first (section 6). When the household disputes a number — “why €1,710 when I earn €2,240?” — the answer is already in the file, step by step.

8. Run it

```
law ask <case package> --case Familie2025 --query-json \
  queries/einkommen-1710.json
law ask <case package> --case Familie2025 --query-json \
  queries/miete-777-80.json
```

The first asks whether the monthly assessed income is €1,710.00, the second whether the rent taken into account is €777.80. Both were **established** on 12 October 2026 with law 0.1.12, and so they are on 31 December 2024 (--case Familie20241231).

Exercise

1. Using the table in section 3, what would Y be if Katrin had proven work expenses of €2,000 a year instead of using the flat rate?
2. The Vogts move to a flat in the same town for €850 a month, without heating. What is M in 2025?
3. Why does the model not assume that a person registered at the address is a household member?

One Family, One Decision: Weimar, Mietenstufe III, €485

On a first read, follow the family's question and answer and come back to the code afterwards. By the end of this chapter you will be able to read one decision from the model: the facts it used, the rules that applied, the sections behind them — and what moves when a fact changes.

Lawyer: which section fixes each step of the answer.

Analyst: which facts the answer depends on, and by how much.

Developer: how a case holds facts and questions, and what comes back.

1. The question

The Vogts from Chapter 0 applied for Wohngeld from January 2025. Chapters 1 and 2 built the pieces: the edition in force, the household, $Y = €1,710.00$ and $M = €777.80$. Now the whole decision, in one place.

Predict before you read on: if the Vogts' rent rose from €580 to €850, would their Wohngeld rise by the full €270, by less, or not at all?

2. How the case is built

A *case* holds the facts of one situation. The rules are not in the case: they come from the *canons* it depends on — reusable packages of legal rules, one per act — the Wohngeld canon, the WoGV canon with the rent levels and the EStG canon with the income definitions. The case records who

the household is, what is known about it, on which date, and where each fact comes from.

```
case Familie2025 {
  context {
    legal_time @2025-01-01;
    knowledge_time @2026-09-15T09:00:00Z;
    decision_time @2026-09-15T09:00:00Z;
    timezone "Europe/Berlin";
  }

  assert "antrag": de.social.wohngeld::wohngeldantrag(...
    haushalt..., ...ehefrau..., de.social.wohngeld::
    MIETZUSCHUSS) { origin case_input; }
  assert "wohnort": de.social.wohngeld::wohnort(...
    haushalt..., "Thüringen", "Weimar, Stadt") { origin
    case_input; }
  assert "entgelt": de.social.wohngeld::vereinbartes_
    entgelt(...mietverhaeltnis..., 580.00 EUR) { origin case_
    input; }
  assert "ehefrau-lohn": de.tax.income_for_wohngeld::
    arbeitslohn_in_geld(...ehefrau..., 26880.00 EUR) { origin
    case_input; }
  assert "behoerde-21-nr3": not de.social.wohngeld::
    missbraeuchliche_inanspruchnahme(...haushalt...) { origin
    adjudicated; }
  ...
}
```

The full case has 42 facts. Three things to notice:

- **The date of the question is part of the case.** `legal_time` selects the edition of the Act and the rows of the annexes: the values in force on 1 January 2025.
- **Every fact says where it comes from.** Most are `case_input` — declared by the household. The assessment under § 21 Nr. 3 is adjudicated — decided by the Wohngeldbehörde. The proof keeps the two apart.
- **The place is a name, not a rent level.** The case says “Weimar, Stadt” in Thuringia; the model reads the rent level from the WoGV’s own list. Nobody types “III”.

3. The decision, step by step

Step	Value	Rule and section
Household	4 members, all counted	§ 5 Abs. 1, §§ 6, 7 WoGG
Annual income of Katrin	€26,880 – €1,230 = €25,650; – 20 % = €20,520	§ 14 WoGG, § 9a EStG, § 16 WoGG
Monthly assessed income Y	€1,710.00	§ 13 Abs. 2 WoGG
Rent level	Mietenstufe III (Weimar, Stadt)	§ 12 Abs. 1 WoGG, Anlage to § 1 Abs. 3 WoGV
Rent under § 9	€580.00	§ 9 WoGG
Ceiling + climate component	€766 + €34.40 = €800.40	§ 12 Abs. 1, 7, Anlage 1
Heating relief	€197.80	§ 12 Abs. 6
Rent taken into account M	min(€580, €800.40) + €197.80 = €777.80	§ 11 Abs. 1
Unrounded Wohngeld z4	484.7809136900	§ 19 Abs. 1, Anlagen 2, 3
Monthly Wohngeld	€485	Anlage 3 Nr. 3
Claim	€485 , established	§§ 19–21 WoGG

Each line is a node in the proof graph, with the rule that produced it and the section it cites. The ministry's *Rechenbeispiel 7* prints the rent level, Y (€1,710.00), the rent (€580), the ceiling with its climate component (€766 + €34.40), the heating relief (€197.80), M (€777.80) and the Wohngeld (€485), and the model matches each of them. The annual figures, z4 and the claim are the model's own: the example works month by month, prints no step of the formula and, as its brochure says, establishes no claim.

4. The answer card

Every volume of this series shows a decision on the same card:

Question	monthly Wohngeld and the claim to it, Familie Vogt
Legal date	1 January 2025 — values of the second WoGFV
Answer	€485 a month (Mietzuschuss) — established
Claim	€485 — established
Decisive provisions	the amount — § 19 Abs. 1–2 with Anlagen 1–3, last step Anlage 3 Nr. 3; the claim — §§ 19–21
Sources	§ 19 Abs. 1, 2 WoGG; Anlagen 1–3 as published in BGBl. 2024 I Nr. 314
Readings applied	intermediate values longer than ten decimal places are rounded half up to ten (Chapter 4)
Open questions	none — the § 21 Nr. 3 assessment is recorded as decided by the authority
Fingerprints	program sha256:8a69e9ad..., case sha256:8eb8b743... (Chapter 9)

5. Change one thing

The same case, one fact changed at a time. Each line is a run of the model, recorded 12 October 2026:

Change	Rent taken into account M, €/month	Assessed income Y, €/month	Monthly Wohngeld
none — the Vogts as they are	€777.80	€1,710.00	€485
the date: 31 December 2024	€777.80	€1,710.00	€463
the rent: €850 instead of €580	€998.20	€1,710.00	€644
income tax also expected (§ 16, three deductions)	€777.80	€1,496.25	€542
proven work expenses €2,000 instead of the flat rate	€777.80	€1,658.6666666667 — held at ten decimal places (rounded half up), not rounded to the cent	€499
Markus not established as living in the flat	€750.20	€1,600.00	€351 , for 3 members
the § 21 Nr. 3 assessment not yet made	€777.80	€1,710.00	€485 — claim <i>requires judgment</i>

Now the prediction. A rent of €850 is above the ceiling plus climate component (€800.40), so only €220.40 more counts, not €270; and the formula pays about 72 cents of each extra euro of M. The Wohngeld rises by €159, to €644. Less than the full increase — and once the rent is above the ceiling, not at all.

The table also shows why the amount alone is not the decision. The last line has the same €485 as the first, and yet no claim is established: the authority has not spoken (Chapter 6).

6. What this gives the authority

One case, one place: the household, the income and rent as the Act defines them, the edition and rows used, the amount, and the claim with every premise. A caseworker can see what is established and what is still open; a team leader can see which fact would change the amount and by how much; a reviewer can follow every number to its section and its gazette.

7. Where Catala stands on this

Catala 1.3.0 supports the exact arithmetic this formula needs — exact decimals, money and dates (Catala book, ch. 5.2³). The difference begins after the number: Arxo returns the decision around it — the claim kept apart from the amount, the authority’s assessment marked as decided by the authority, and each annex row tied to its gazette issue.

8. Run it

```
law ask <case package> --case Familie2025 --query-json \
queries/einkommen-1710.json
law ask <case package> --case Familie2025 --query-json \
queries/miete-777-80.json
law ask <case package> --case Familie2025 --query-json \
queries/wohngeld-485.json
law ask <case package> --case Familie2025 --query-json \
queries/anspruch-485.json
```

All four answers are *established*, recorded on 12 October 2026 with law 0.1.12. The 31 December 2024 and the pending-assessment lines of section 5 are the cases Familie20241231 and FamilieOhneWuerdigung of the same package; the other four variations — rent, income tax, work expenses and Markus — are cases of a second case package, familie-weimar-varianten, listed in the appendix on reproducing the examples.

Exercise

1. Which fact of the case would you change to move the Vogts from Mietenstufe III to another rent level, and why can you not simply enter “IV”?
2. In section 5, the rent rises by €270 but M rises by only €220.40. Which section caps it?
3. The answer card lists “open questions: none”. Which fact of the case makes that true?

³<https://book.catala-lang.org/en/5-2-types.html>

The § 19 Formula: Anlagen 2 and 3 and One Rounding

By the end of this chapter you will be able to compute the Vogts' €485 by hand, step by step, and say where the Act fixes the arithmetic and where the model had to choose.

Lawyer: which provision fixes the order of the steps and the rounding.
Analyst: how sensitive the amount is to M and Y .
Developer: which four nodes of the proof carry the formula, and how z_1 to z_3 are recomputed from them.

1. The question

You know the three inputs from Chapter 2: four household members, $M = €777.80$, $Y = €1,710.00$. How do they become €485?

Predict before you read on: in how many places does the Act let the calculation round — everywhere, once at the end, or nowhere?

2. The formula as the Act writes it

§ 19 Abs. 1 WoGG:

Das ungerundete monatliche Wohngeld für bis zu zwölf zu berücksichtigende Haushaltsmitglieder beträgt $1,15 \cdot (M - (a + b \cdot M + c \cdot Y) \cdot Y)$ Euro. „ M “ ist die zu berücksichtigende monatliche Miete oder Belastung in Euro. „ Y “ ist das monatliche Gesamteinkommen in Euro. „ a “, „ b “ und „ c “ sind nach der

Anzahl der zu berücksichtigenden Haushaltsmitglieder unterschiedene Werte und ergeben sich aus der Anlage 2.

In English: the unrounded monthly Wohngeld for up to twelve counted members is $1.15 \cdot (M - (a + b \cdot M + c \cdot Y) \cdot Y)$ euros, with a , b and c taken from Anlage 2 by the number of members.

§ 19 Abs. 2 adds that the steps and roundings must be carried out in the order set out in Anlage 3. Anlage 3 is short and exact:

1. values of M and Y below the minimums in its table are replaced by those minimums;
2. the formula is computed in four intermediate steps, $z1$ to $z4$, “als Festkommazahlen mit zehn Nachkommastellen” — as fixed-point numbers with ten decimal places;
3. the result is rounded to the full euro: below 50 cents down, from 50 cents up.

So the answer to the prediction: there is one rounding to the full euro, at the end. The intermediate results are held at ten decimal places; how a longer intermediate result is brought to ten places is the model’s choice, explained in section 5.

3. The Vogts, step by step

For four members, the second WoGFV sets $a = 0.01$, $b = 2.163 \times 10^{-4}$ and $c = 1.760 \times 10^{-5}$, and the minimums $M = \text{€}92$ and $Y = \text{€}1,132$. The Vogts are above both minimums, so step 1 changes nothing.

Step	Calculation	Value
$z1$	$a + b \cdot M + c \cdot Y = 0.01 + 0.0002163 \cdot 777.80 + 0.0000176 \cdot 1,710$	0.2083341400
$z2$	$z1 \cdot Y$	356.2513794000
$z3$	$M - z2$	421.5486206000
$z4$	$1.15 \cdot z3$	484.7809136900
rounding	484.78... to the full euro	€485

The proof of the model’s answer contains the same $z4$ — 484.78091369, trailing zeros dropped — on the node of the rule *WohngeldUngerundet*,

and the rounded €485 on the node *WohngeldGerundetBisZwoelf*. On 31 December 2024, with the values of the *Wohngeld-Plus-Gesetz* ($b = 2.251 \times 10^{-4}$, $c = 2.000 \times 10^{-5}$), the same steps give $z4 = 463.2504131300$ and €463.

Read the table backwards and you see the logic of the benefit: $z2$ is the part of the rent the household is expected to carry itself, growing with income; *Wohngeld* pays 115 % of the rest.

4. The steps in the model

The model writes the three numbers of Anlage 3 as rules: Nr. 1, the minimums of M and Y , as two rules; Nr. 2 as one rule whose function computes $z1$, $z2$ and $z3$ as intermediate values and records $z4$; and Nr. 3, the rounding to the euro, as a rule of its own. The proof therefore shows four nodes for the formula — M , Y , $z4$ and the rounded amount — and $z1$ to $z3$ are recomputed from them, as in the table above:

```
function z4_berechnen(k: Integer, m: Decimal, y: Decimal) ->
  Decimal {
    let z1: Decimal = round(wert_a(k) + wert_b(k) * m +
      wert_c(k) * y, 10, "HALF_UP");
    let z2: Decimal = round(z1 * y, 10, "HALF_UP");
    let z3: Decimal = m - z2;
    return round(FormelFaktor * z3, 10, "HALF_UP");
  }

rule WohngeldGerundetBisZwoelf(h: Haushalt, z4: Decimal, n:
  Integer) strict {
  label de official "Anlage 3 Nummer 3: bis unter 0,50
    Euro auf den nächsten vollen Euro-Betrag abzurunden,
    von 0,50 Euro an aufzurunden";
  when wohngeld_ungerundet(h, z4) and anzahl_zu_
    beruecksichtigende_haushaltsmitglieder(h, n) and n <=
    12;
  then wohngeld_monatlich(h, 1.00 EUR * round(z4, 0,
    "HALF_UP"));
}
```

Three things to notice:

- **The arithmetic is exact.** No binary floating point: decimals stay decimals, and each intermediate value is held at exactly the ten places the Anlage asks for.

- **The rounding to the euro is where the Act puts it** — one rule, labelled with the words of Anlage 3 Nr. 3, after `z4` and nowhere else.
- **a, b and c are parameters, not constants in the code.** `wert_b(k)` reads the row of Anlage 2 in force on the date of the question; that is how the same function gives €463 and €485 in Chapter 1.

5. The one choice the text leaves open

Anlage 3 says “mit zehn Nachkommastellen zu berechnen” — compute with ten decimal places. It does not say how a longer intermediate result is brought to ten places: rounded or cut off. For the products in `z1` and `z2` this can matter in the last place.

The model takes commercial rounding (half up) — the method Anlage 3 Nr. 3 prescribes for the final step, and § 43 Abs. 4 and 7–9 for the amounts in euros of each update. The Act is not uniform, though: for the values `b` and `c`, § 43 Abs. 5 and 6 order them to be rounded down at the seventh decimal place. So the text gives no single answer for the intermediate values `z1` to `z4`; the model chooses half up and documents the choice beside the function. When does it matter? Different methods for the intermediate steps can move the final euro only when `z4` lies very close to a half euro. For the Vogts, `z4` = 484.78..., far from 484.50; both methods give €485. A reader who prefers the other method can see exactly which line to change and which cases it could affect.

6. Change one thing

How strongly do `M` and `Y` move the amount? Two variations of the Vogts’ case, run through the model (Chapter 3 shows them in full):

- **Higher rent.** At a rent of €850, `M` rises by €220.40 to the ceiling’s €998.20, and the Wohngeld from €485 to **€644** — about 72 cents per euro of `M`.
- **Lower income.** If income tax were also expected, § 16 would deduct a third 10 %; `Y` falls by €213.75 to €1,496.25, and the Wohngeld rises to **€542** — about 27 cents per euro of `Y`.

The formula is built that way: in the Vogts’ range, each euro of rent taken into account raises the benefit far more than each euro of income lowers it. Neither variation needs a new formula — only a new fact.

7. What this gives the authority

When a household asks “why €485 and not €490?”, the caseworker can show M, Y and z4 from the proof with the values of the right Anlage, recompute z1 to z3 from them, and point to the one rounding the Act allows. When the ministry publishes a worked example, the office can compare it step by step, not only at the last line. And where the model made a choice the text does not make, the choice is written down where a reviewer will find it.

8. Where Catala stands on this

Catala supports exact rational decimals and gives decimal-to-money conversion a defined rounding to the nearest cent (*Catala book*, ch. 5.2⁴). That is a clear computational convention, and for a formula like § 19 it gives exact intermediate values too. Arxo adds that each of the three numbers of Anlage 3 is a rule, tied to the rows of the Anlagen it read: M, Y, z4 and the single euro rounding can be checked in the answer itself, and z1 to z3 recomputed by hand from M and Y.

9. Run it

```
law ask <case package> --case Familie2025 --query-json \
  queries/wohngeld-485.json --out runs/familie-2025
```

In `runs/familie-2025/result.json`, find the proof node whose rule is *WohngeldUngerundet*. Its conclusion carries z4. Its attribute `a3Selections` names the rows of Anlage 2 it read, and the factor 1.15:

```
WertAMitglieder4#2025-01-01
WertBMitglieder4#2025-01-01
WertCMitglieder4#2025-01-01
FormelFaktor#2023-01-01
```

Recorded on 12 October 2026 with law 0.1.12.

⁴<https://book.catala-lang.org/en/5-2-types.html>

Exercise

1. Using the values of section 3, compute z_1 for 31 December 2024 ($a = 0.01$, $b = 2.251 \times 10^{-4}$, $c = 2.000 \times 10^{-5}$, $M = 777.80$, $Y = 1,710$).
2. A colleague rounds z_1 and z_2 to the cent “to keep it simple”. Why does the model not do that?
3. In which single situation could the model’s reading of “zehn Nachkommastellen” and the alternative reading give different amounts?

How a Scenario Is Written

By the end you can read one scenario, say where its expected numbers come from, and explain what its success establishes and what it does not.

Lawyer: whether the expectation follows the published text or example.

Analyst: which edge or example the scenario exercises.

Developer: facts, question, expectation and the declared world.

1. The question

How do we know the model's €485 is right? Not because the model says so. We know because someone outside the model — the federal ministry — published the same family and the same answer, and a scenario holds the model to it.

2. Where the expectation comes from

The BMWWSB publishes worked examples of the Wohngeld calculation: ten for 2023 and eleven for 2025. Each gives a household, its rent and income, the intermediate values and the monthly amount. The model's scenarios take their **expected numbers from these PDFs**, not from a run of the engine. A scenario that was written by copying the engine's output would test nothing.

Rechenbeispiel 7 of 2025 — the Vogts' example — gives $Y = €1,710.00$, $M = €777.80$, the ceiling plus climate component €800.40, and Wohngeld €485. The ministry writes the income in one line: €2,240 – €102.50

= €2,137.50; minus 20 % = €1,710.00. The rent: €580 + €197.80 = €777.80.

Before each scenario stands a comment saying which PDF, which example, and which assumptions the case had to make where the PDF is silent. For example 7: the PDF does not give the children's ages; the case enters 10 and 6, which do not affect the amount.

3. Facts, question, expectation

A scenario has three parts. Here is the shape of the one for example 7, shortened:

```
test "BMWSB 2025 Rechenbeispiel 7: Ehepaar mit zwei Kindern,
    Weimar (Mietenstufe III) – 485 EUR" {
  given {
    context { legal_time @2025-01-01; timezone "Europe/
      Berlin"; ... }
    assert "wohnort": wohnort(haushalt, "Thüringen",
      "Weimar, Stadt") { origin case_input; }
    assert "entgelt": vereinbartes_
      entgelt(mietverhaeltnis, 580.00 EUR) { origin
        case_input; }
    assert "ehefrau-lohn": arbeitslohn_in_geld(ehefrau,
      26880.00 EUR) { origin case_input; }
    assert "ehefrau-abzug16": abzugsmerkmale_16(ehefrau,
      NEIN, JA, JA) { origin case_input; }
    assert "behoerde-21-nr3": not missbraeuchliche_
      inanspruchnahme(haushalt) { origin adjudicated; }
    ...
  }
  evaluate truth(monatliches_gesamteinkommen(haushalt,
    1710.00 EUR));
  expect truth_status == TRUE_ONLY;
  evaluate truth(zu_beruecksichtigende_miete(haushalt,
    777.80 EUR));
  expect truth_status == TRUE_ONLY;
  evaluate truth(hochstbetrag_mit_
    klimakomponente(haushalt, 800.40 EUR));
  expect truth_status == TRUE_ONLY;
  evaluate truth(wohngeld_monatlich(haushalt, 485 EUR));
  expect truth_status == TRUE_ONLY;
  evaluate truth(wohngeldanspruch(haushalt, 485 EUR));
  expect truth_status == TRUE_ONLY;
  expect judgment_requests(0);
}
```

- **given** holds the facts, each with its origin. Most are `case_input` — what the household declared. One is `adjudicated`: the authority's own assessment under § 21 Nr. 3 (Chapter 6).
- **evaluate** asks a question of the model. Here, five questions: the intermediate values the PDF prints, the amount and the claim.
- **expect** states the answer the source requires. `TRUE_ONLY` means *established*. The expected income, rent, ceiling and amount come from the PDF. The expected claim does not: the ministry's examples expressly establish no claim. The scenario adds the authority's assessment under § 21 Nr. 3 as a decided fact (`adjudicated`), and only with it does the claim follow; `judgment_requests(0)` then says nothing is left for the authority.

The scenario checks the intermediate values, not only the last line. A model that reached €485 by two errors that cancel out would fail at M or Y.

4. The declared world

A scenario runs against a declared set of canons — its *world*: here the Wohngeld canon and the two it reads, the ordinance with the rent levels and the income definitions of the EStG. The world is part of the test. A scenario that passed against a different set of rules would prove something else.

5. What the scenarios cover today

Source	Examples	In the model	Outside the model
BMWSB worked examples 2023	10	9 tenant examples	1 — Lastenzuschuss
BMWSB worked examples 2025	11	9 tenant examples	2 — Lastenzuschuss

18 of 21 published examples are executed; the three outside the model are owner-occupier subsidies, which this model does not cover (Chapter 8). Next to the published examples stand scenarios for the change of edition, missing and conflicting facts, exclusions and the authority's judgment — the suites Chapters 1, 6, 7 and 8 draw on.

On 12 October 2026, with law 0.1.12, the 2023 suite passed 9 of 9 scenarios and the 2025 suite 10 of 10 — one more scenario than published tenant examples, because example 1 is run twice, with two assumptions about the taxable share of the pension that the PDF does not state; the edition suite 6 of 6; the suite of missing and conflicting facts 7 of 7; the suite of exclusions and judgments 16 of 16.

6. A case package is a scenario you can ask

The scenarios test the canon. The *case package* familie-weimar — the one this book asks — holds the same family as a set of standing files: four cases (Familie2025, Familie20241231, FamilieOhneWuerdigung, AlleinGrundsicherung) and six saved questions. Its check script asks eleven case-question pairs and verifies, for each:

- the status of the answer and any request for the authority’s judgment;
- that the decisive rule applied, with its anchor in the edition of the date;
- which row of Anlage 2 the formula used, and that row’s anchor to the BGBl issue;
- that a replay of the saved answer gives the same bytes.

7. What a passing scenario establishes — and what it does not

A passing scenario establishes that the model, with these facts and this world, gives the answer the published example gives — including its intermediate values. It does **not** establish that the published example is right, that the model covers every case the Act covers, or that a particular office applies the Act the same way. Agreement with the ministry’s examples is strong evidence about the formula; it is not a legal opinion.

8. Where Catala stands on this

Catala has a mature testing workflow: `clerk test` discovers scope tests and command-output tests, with backend checks available (Catala book, ch. 3.3⁵). This is shared ground: both approaches preserve examples and expectations. Arxo adds built-in query statuses — established, not established, contradictory, waiting for an authority — so a scenario

⁵<https://book.catala-lang.org/en/3-3-0-test-ci.html>

can expect “not established” or “the Wohngeldbehörde must assess” as precisely as it expects €485.

9. Run the scenarios

```
law test <canon> --file tests/bmwsb-2025.lawtest
law test <canon> --file tests/redaktionen.lawtest
```

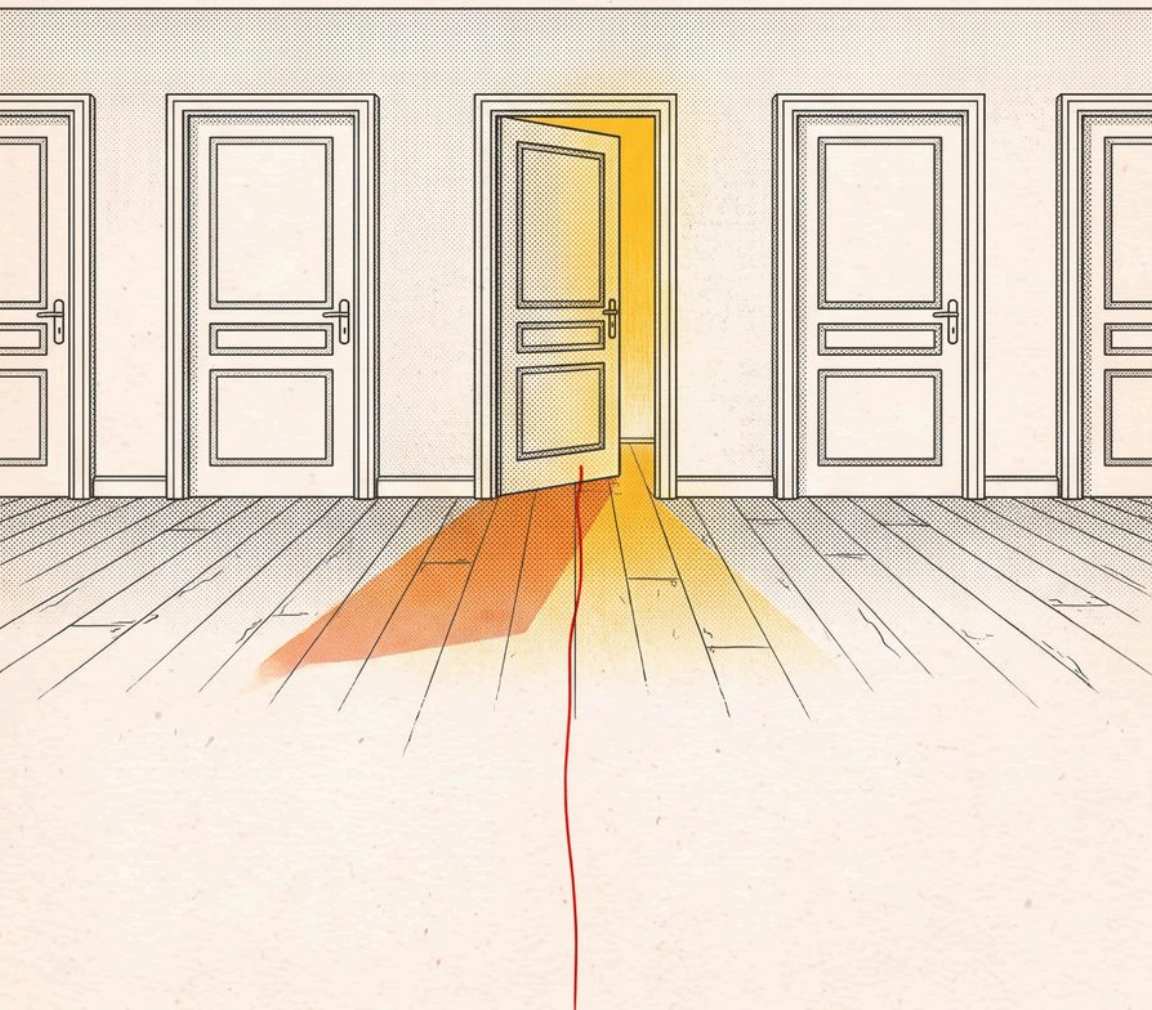
Each line reports every scenario as passed or failed, and a total.

Exercise

1. Why must the expected €485 come from the ministry’s PDF and not from the model?
2. The scenario for example 7 also expects $M = €777.80$ and the ceiling €800.40. What kind of error would these extra expectations catch that the final €485 alone would not?
3. Three of the 21 published examples are not executed. Is that a failure of the scenarios?

PART III

Where the Text Is Hard



Sum Known, Claim Pending: § 21 Nr. 3 and the Wohngeldbehörde

By the end of this chapter you will be able to tell an amount from a claim, recognise a question the Act leaves to the authority, and read the request the model addresses to it.

Lawyer: which words of § 21 Nr. 3 reserve the judgment, and for whom.

Analyst: which answers change when the authority answers.

Developer: what a request for judgment looks like in the answer.

1. The question

The Vogts' file arrives at the Wohngeldbehörde in Weimar. Everything is complete — household, rent, income — except one thing: the office has not yet assessed whether claiming Wohngeld would be abusive.

Predict before you read on. What should a model say about the claim to €485 now: established, not established, or something else?

2. The text

§ 21 WoGG lists three cases in which no claim exists:

Ein Wohngeldanspruch besteht nicht, 1. wenn das Wohngeld weniger als 10 Euro monatlich betragen würde, 2. wenn alle Haushaltsmitglieder nach den §§ 7 und 8 Abs. 1 vom Wohngeld ausgeschlossen sind oder 3. soweit

die Inanspruchnahme missbräuchlich wäre, insbesondere wegen erheblichen Vermögens.

In English: no claim exists (1) if the Wohngeld would be under €10 a month, (2) if all household members are excluded, or (3) to the extent that claiming it would be abusive, in particular because of considerable assets.

Numbers 1 and 2 are tests the model can run. Number 3 is different. “Missbräuchlich” — abusive — is an evaluative word. The Act gives no threshold for “considerable assets”; the office must weigh the circumstances of the case. The model leaves that weighing to the Wohngeldbehörde rather than to a formula — a choice about where its automation stops.

3. How the model holds the judgment

The model declares the judgment as a relation that, in this model, only an authority can settle:

```
pub external judgment relation missbraeuchliche_
  inanspruchnahme(h: Haushalt) {
    authority Wohngeldbehoerde;
    label de official "die Inanspruchnahme von Wohngeld wäre
      missbräuchlich, insbesondere wegen erheblichen
      Vermögens (§ 21 Nummer 3)";
  }
```

and the claim requires, among other premises, that it does *not* hold:

```
rule Wohngeldanspruch(h: Haushalt, p: Person, w: Money)
  strict {
    when wohngeldberechtigt(h, p) and wohngeld_monatlich(h,
      w) and w >= 10 EUR
      and kein_ausschluss_nach_20(h)
      and not missbraeuchliche_inanspruchnahme(h);
    then wohngeldanspruch(h, w);
  }
```

One of the other premises, `kein_ausschluss_nach_20`, is § 20 Abs. 2: no claim exists if every member of the household is eligible in principle

for one of the kinds of training support it lists, such as BAföG, unless that support is granted only as a loan. The Vogts’ case records, as a declared fact (bafoeg, origin case_input), that not all of them are.

The amount (§ 19) and the claim (§§ 19–21) are two different conclusions. The ministry’s own brochure says the same: its examples show the amount by the formula and expressly “begründen keinen Rechtsanspruch” — establish no legal claim.

4. The answers

The case FamilieOhneWuerdigung is the Vogts on 1 January 2025 with every fact of Familie2025 — except the authority’s assessment under § 21 Nr. 3.

Question	Familie2025 (assessed)	FamilieOhneWuerdigung (not yet assessed)
Monthly assessed income €1,710.00	established	established
Rent taken into account €777.80	established	established
Monthly Wohngeld €485	established	established
Claim to €485	established	not established — an assessment by the authority is required

That answers the prediction: *something else*. The amount is known; the claim waits. Technically the answer has two parts: the claim is *not established* (truth status NEITHER), and the evaluation is waiting for an authority (evaluation status REQUIRES_JUDGMENT), with a request:

```
{
  "authority": "urn:de:social:clir:
    wohngeld#Wohngeldbehoerde",
  "predicate": "urn:de:social:clir:
    wohngeld#missbraeuchliche_inanspruchnahme",
  "request": {
    "relation": "urn:de:social:clir:
      wohngeld#missbraeuchliche_inanspruchnahme",
    "polarity": "negative",
    "args": [{ "kind": "entity_ref", "id": "urn:de:example:
      wohngeld:familie-weimar:haushalt" }]
  }
}
```

It names who must answer (the Wohngeldbehörde), what (whether claiming would be abusive, for this household), and which answer the claim needs (negative — that it would *not* be).

5. The authority answers

In Familie2025 the office has answered. The case records its assessment with a different origin from the household's own data:

```
assert "behoerde-21-nr3": not de.social.wohngeld::
  missbraeuchliche_inanspruchnahme(...haushalt...) { origin
  adjudicated; }
```

adjudicated — decided by the competent authority — not case_input. The proof shows which of the two the claim rests on.

The suite of exclusions and judgments runs all three states on a second household (a single person in Weimar, wages €18,000 a year, rent €500, on 1 March 2025):

The office has ...	Monthly Wohngeld	Claim
not yet assessed	€202, established	requires judgment, request to the Wohngeldbehörde
found no abuse	€202, established	€202, established
found abuse	€202, established	no claim — ground § 21 Nr. 3, established

The amount never moves. Only the authority's answer moves the claim.

6. Why this matters

For the household. A missing assessment is not a suspicion. The model does not treat silence as “abusive”, and it does not treat silence as “not abusive” either. It says the office has not decided yet.

For the office. Wohngeld is social law, decided under the SGB X: § 31a SGB X allows a decision to be issued fully automatically only where there is no reason to have an official handle the individual case. The general rule of administrative procedure, § 35a VwVfG, which excludes discretion and margins of appreciation from full automation, is named here only for comparison. A request for judgment marks, in the answer itself, the point where this model stops automating and leaves the question to a person of the authority. This is the model’s choice, not a legal qualification: whether a given word of the Act gives the office a margin of appreciation is a question for lawyers and courts. The model makes the point visible; it does not decide it.

For the system. A calculated amount does not establish entitlement by itself. A calculator that silently assumes “not abusive” presents a claim where the authority has not yet spoken.

7. A second place the Act asks for judgment

§ 5 Abs. 1 S. 2 Nr. 3 counts as a household member someone who lives with a member so that “nach verständiger Würdigung” a mutual will to take responsibility for each other must be assumed. The model holds this the same way: when the facts do not settle it, the answer asks the Wohngeldbehörde. § 5 Abs. 2 adds a presumption — for example, living together for more than a year — which the model applies as a rebuttable rule. The scenarios run all three paths: no presumption and no assessment (judgment requested), the office affirms the community (two members), the presumption applies and is not rebutted (member without a separate assessment).

8. Where Catala stands on this

Catala’s exceptions and defaults let a programme state a general rule and its exceptions beside the text — a strong foundation for conditions like § 21. In Catala 1.3.0, a value such as “abusive” is an input supplied by the

caller; there is no built-in form for a pending decision or its addressee. Arxo builds the request into the answer: the amount is computed, the claim is held, and the authority is named.

9. Run it

```
law ask <case package> --case FamilieOhneWuerdigung \
    --query-json queries/wohngeld-485.json
law ask <case package> --case FamilieOhneWuerdigung \
    --query-json queries/anspruch-485.json
```

The first answer is *established*; the second has the evaluation status `REQUIRES_JUDGMENT` and one entry in `judgmentRequests`. Recorded on 12 October 2026 with law 0.1.12.

Exercise

1. In `FamilieOhneWuerdigung`, which of the four questions in section 4 would change if the office found the claim abusive, and to what?
2. Why does the model not treat the absence of an assessment as “not abusive”, given that most households are not?
3. A colleague proposes to replace § 21 Nr. 3 by a rule “assets above €60,000 = abusive”. What would the model then claim that the Act does not say?

Excluded by Another Authority's Decision: § 7 WoGG

By the end of this chapter you will be able to say when another benefit excludes a person from Wohngeld, what the model needs to know about that benefit, and when the exclusion does not apply.

Lawyer: which benefits exclude, and under which condition.

Analyst: which single fact about another benefit flips the answer.

Developer: how a decision of another authority enters a case.

1. The question

A woman lives alone in a rented flat in Weimar, rent €500. In March 2025 she receives basic income support from the job centre under the Second Book of the Social Code (SGB II), and the job centre's calculation includes her housing costs. She applies for Wohngeld as well.

Predict before you read on: what does the model answer — a reduced Wohngeld, no Wohngeld, or “not enough facts”?

2. The text

§ 7 Abs. 1 Satz 1 WoGG excludes recipients of a list of benefits:

Vom Wohngeld ausgeschlossen sind Empfänger und Empfängerinnen von 1. Bürgergeld nach dem Zweiten Buch Sozialgesetzbuch ... 5. Grundsicherung im Alter und bei Erwerbsminderung nach dem Zwölften Buch Sozialgesetzbuch, 6. Hilfe zum Lebensunterhalt nach dem Zwölften Buch Sozialgesetzbuch ...

wenn bei deren Berechnung Kosten der Unterkunft berücksichtigt worden sind (Leistungen).

In English: recipients of basic income support under SGB II, of basic support in old age and on reduced earning capacity, of subsistence assistance under SGB XII and of six further benefits are excluded from Wohngeld — **if housing costs were taken into account in calculating those benefits**. The other benefit already pays for housing; Wohngeld would pay twice.

A note on names: in March 2025 the SGB II benefit was called *Bürgergeld*. Since 1 July 2026 it is called *Grundsicherungsgeld* (BGBl. 2026 I Nr. 107); the model holds both editions of § 7 and quotes the text in force on the date. The name of the benefit inside the model, NR1_GRUNDSICHERUNGSGELD_SGB2, follows the current edition; for March 2025 the quoted text still says Bürgergeld.

§ 7 Abs. 1 Satz 3 then lifts the exclusion in two situations: when the benefits are granted **only as a loan**, and when Wohngeld can **avoid or end the need for help** and the other benefit has not yet been paid while its procedure runs, or is paid by a carrier only obliged in second rank (§ 104 SGB X). § 8 Abs. 2 adds a third: if a member **waives** the other benefit in order to apply for Wohngeld, the exclusion is treated as not having happened.

3. A fact decided elsewhere

Whether the woman receives SGB II support with housing costs is not a question the Wohngeldbehörde decides. The job centre decided it, in its own decision notice. The model therefore does not infer it; it takes it as a fact, with every detail § 7 needs. `origin case_input` says how the fact entered the case — from the file; the document behind it, the job centre's notice with its date, belongs in the file and should be cited beside the fact:

```
assert "ehefrau-leistung7": de.social.wohngeld::
  leistungsbezug_7(
    entity_ref("...:ehefrau"),
    de.social.wohngeld::NR1_GRUNDSICHERUNGSGELD_SGB2,    //
      which benefit
    de.social.wohngeld::JA,      // housing costs taken into
      account
    de.social.wohngeld::NEIN,    // only as a loan
    de.social.wohngeld::NEIN,    // Wohngeld would avoid the
      need for help (§ 7 Abs. 1 S. 3 Nr. 2)
    de.social.wohngeld::NEIN    // waived under § 8 Abs. 2
  ) { origin case_input; }
```

The value marked “Wohngeld would avoid the need for help” says more than its comment: JA there certifies the whole of § 7 Abs. 1 S. 3 Nr. 2 — that Wohngeld can avoid or end the need for help *and* that alternative a) or b) applies. A case must not enter JA on the first half alone.

Five values in one fact, because § 7 turns on all five. For the Vogts, the same fact says KEINE — no such benefit — for each of the four members. That is a fact too: Chapter 8 shows that when it is missing, the household cannot be established.

4. The answers

The case AlleinGrundsicherung holds this woman’s file on 1 March 2025.

Question	Model’s answer (recorded 12 October 2026)
No claim because all household members are excluded (§ 21 Nr. 2)	established
Monthly Wohngeld €485 (or any amount)	not established
Claim to €485	not established

The path in the proof: the rule *AusschlussEmpfaengerMitKostenDerUnterkunft* (§ 7 Abs. 1 S. 1 and 2) excludes her; with her excluded, the household has no counted member; the rule *AlleHaushaltsmitgliederAusgeschlossen* (§ 21 Nr. 2) concludes that no claim exists.

That answers the prediction: no Wohngeld — and the reason is not “not enough facts” but a ground established by law, resting on the job centre’s decision.

5. Change one thing

The suite of exclusions runs the exceptions, one fact at a time, all passing on 12 October 2026:

Change to the fact about the other benefit	Model's answer
housing costs not taken into account	no exclusion — Wohngeld is computed
benefit granted only as a loan (§ 7 Abs. 1 S. 3 Nr. 1)	no exclusion
benefit waived to apply for Wohngeld (§ 8 Abs. 2)	exclusion treated as not having happened
the excluded person is the applicant, but lives with a member who is not excluded (§ 3 Abs. 4)	the applicant stays entitled to claim for the household

The last line matters in families. In the ministry’s *Rechenbeispiel 10* of 2025, a retired father lives with a son who receives SGB II support with housing costs. The son is excluded; the father is not. The Wohngeld is computed for one counted member out of two, with the rent and its components taken in proportion (§ 11 Abs. 3): €191 a month — the ministry’s number, matched by the scenario.

6. What this gives the authority

The office can see at once which fact excluded a person, which authority’s decision that fact rests on, and which of the exceptions were checked. If the job centre changes its decision — housing costs no longer included, or the benefit turned into a loan — one fact changes, and the answer follows with its new ground. The two decisions stay connected in the file instead of in a caseworker’s memory.

7. Where Catala stands on this

Catala’s exception mechanism is a natural fit for § 7: a general exclusion and its exceptions, written beside the text (*Catala book, ch. 2.2*⁶).

⁶<https://book.catala-lang.org/en/2-2-conditionals-exceptions.html>

GETTSIM goes further on the economics, computing which of the competing transfers a household should take. What Arxo adds in this chapter is the status of the input: the decision of another authority enters as a fact with its origin, and the excluding ground in the answer names the provision that turned it into “no claim”.

8. Run it

```
law ask <case package> --case AlleinGrundsicherung \
  --query-json \
  queries/kein-anspruch-alle-ausgeschlossen.json
law ask <case package> --case AlleinGrundsicherung \
  --query-json queries/wohngeld-485.json
```

The first answer is *established*; the second *not established*. The exceptions are scenarios of the canon: `law test <canon> --file tests/ausschluss-und-urteil.lawtest` (16 of 16 passed on 12 October 2026 with law 0.1.12).

Exercise

1. The woman’s job centre grants her SGB II support only as a loan. What changes in the model’s answer, and under which provision?
2. Why does the model need to know whether housing costs were taken into account, and not only whether the woman receives SGB II support?
3. In *Rechenbeispiel 10*, why does the father keep a claim although his son is excluded?

Missing Facts, Conflicting Records and the Model's Boundary

By the end of this chapter you will be able to tell four answers apart that a calculator easily confuses: “not enough facts”, “the records disagree”, “outside the model” and “past the model's values”.

Lawyer: which provision makes the missing fact a premise.

Analyst: which fact to request from the household.

Developer: which status or issue the answer carries in each case.

1. The question

Files are rarely complete on the first day. The Vogts' application arrives without the address of the flat. A week later a second rental record appears with a different rent. And a neighbour who owns her flat asks the same office about Wohngeld.

Predict before you read on: for the Vogts' file without the address, should a model return €0, an error, or something else?

2. Not enough facts: not established, not zero

The scenarios of the suite of missing and conflicting facts take the Vogts' case and remove one fact at a time:

Fact removed	What the model still establishes	What it does not
the place of residence	monthly income €1,710.00	the rent level, so no ceiling and no amount — the answer to “€0?” is <i>not established</i>
the agreed rent	the rent level, Mietenstufe III	the rent taken into account, so no amount
whether Markus receives a benefit under § 7	—	whether the household is established, so no count of members and no amount
the work expenses behind Katrin’s wages	that her income details are incomplete	the assessed income, so no amount
how a co-resident relates to the family	—	the household, so no amount

The answer to the prediction is the first line. Without the place, the model cannot know the rent level, so it cannot know the ceiling, so it cannot compute M — and it says exactly that: the amount is *not established*. It does not return €0, which would state that the family is owed nothing, and it does not fail: it computes everything that can be computed, the income included.

The third line deserves a second look. A missing fact about another benefit is not the fact “receives no other benefit”. § 7 makes the absence of an excluding benefit a premise; a model that assumed it would pay Wohngeld to people the Act excludes.

Not every missing fact stops the calculation, and the difference is instructive. Remove the fact that the flat is the centre of Markus’s life, and the model does not stop: § 5 Abs. 1 makes that fact the condition of membership, so without it Markus is not counted as a member — although nobody has found that he lives elsewhere. The model computes a household of three — Katrin as a single parent with the allowance of § 17 Nr. 3 — and gives **€351** instead of €485 (Chapter 2). The model does not flag the missing fact. €351 is the result of an incomplete file, not the Vogts’ answer if Markus does live there, and it must not be given to the family as their answer until the household’s composition is established. Where a fact *defines* who is in the household, the caseworker must establish it before reading the amount. The proof shows the count

of members and the rule that produced it, so the difference is visible at the first line.

3. Records that disagree

Two different situations, and the model keeps them apart.

Two values where the model admits one. The rent agreed for one tenancy has one amount; the model declares it so. If the case holds two different agreed rents for the same tenancy — €580 and €620, say — the answer carries the issue `KEY_CONFLICT`. The model does not pick one; it reports that the file contradicts itself on a value that must be unique.

A fact asserted and denied. If one record says the flat is the centre of Markus's life and another denies it, the answer to that question is `BOTH` — asserted and denied at once. The contradiction is kept, with both sources, for the caseworker to resolve. It is not silently settled by whichever record was read last.

4. Outside the model

The neighbour owns her flat. Owner-occupiers receive a *Lastenzuschuss*, governed by § 10 WoGG and Part 3 of the WoGV. This model covers tenants only. For her application it returns:

Question	Answer
Is the case outside the model, because it is a Lastenzuschuss?	established
Is the monthly Wohngeld €320 (the ministry's figure for a similar example)?	not established

The flag `ausserhalb_des_modells` says why there is no number. “This model does not cover owner-occupiers” is a statement about the model. “The law gives you nothing” would be a statement about the law, and a false one. The model keeps the two apart. The same flag marks the other parts of the Act the model leaves out — among them some co-residents whose share of the rent § 11 Abs. 2 sets aside, and a residence title under § 3 Abs. 5 Satz 3.

5. Past the model’s values

Chapter 1 met the fourth answer. Ask the Vogts’ question for 1 January 2027:

Question	Answer (recorded 12 October 2026)
Monthly Wohngeld on 1 January 2027	no amount, with the issue MISSING_PARAMETER_VALUE

The case is complete and inside the model; what is missing is annex values for that date in the model, and the issue names them. Whether values are in force on that date is a question about the law; the issue says only that this model holds none.

6. Four answers side by side

Situation	Status or issue	What the office should do
a premise is missing	<i>not established</i>	ask the household for the missing fact
records contradict	KEY_CONFLICT or BOTH	resolve the records
the case is outside the model	the flag <i>ausserhalb_des_modells</i> , <i>established</i>	decide the case by other means
the date is past the last values the model holds	MISSING_PARAMETER_VALUE	wait for, or model, the new values

Each of these stops the office from reading a number as the answer. Each tells the office something different to do.

7. Where Catala stands on this

Catala 1.3.0 has optional values — absent or present — built in (Catala book, ch. 5.2⁷); the caller decides what to pass for an unknown fact. When two applicable definitions are not exceptions to each other, evaluation stops with a conflict error (ch. 2.2⁸) — a sound guard for the program’s own rules. Arxo adds statuses for the facts of a case: a missing premise yields *not established*, contradictory records yield BOTH kept for review, and a case outside the model’s scope yields a flag that names the reason.

⁷<https://book.catala-lang.org/en/5-2-types.html>

⁸<https://book.catala-lang.org/en/2-2-conditionals-exceptions.html>

8. Run it

```
law test <canon> --file tests/tatsachen.lawtest
law test <canon> --file \
    tests/ausschluss-und-urteil.lawtest
law test <canon> --file tests/redaktionen.lawtest
```

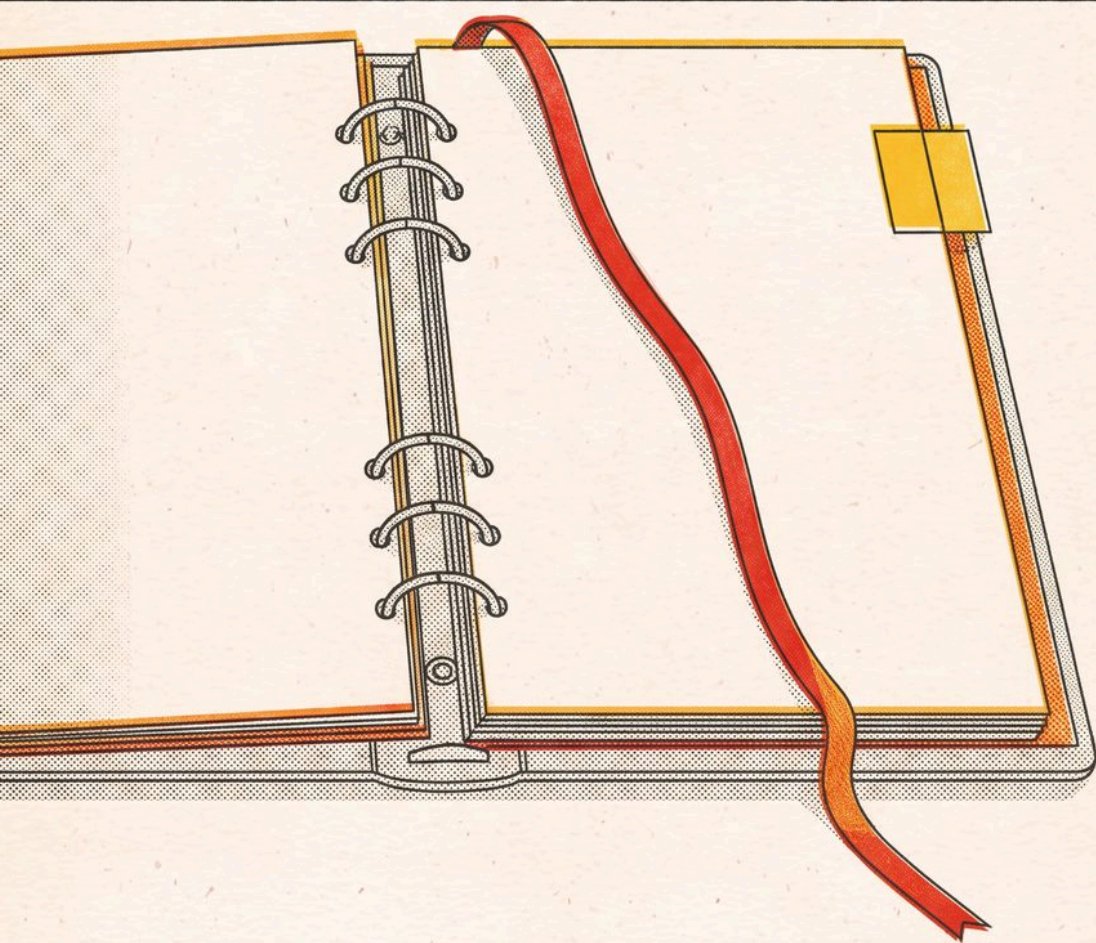
The first suite holds the missing and conflicting facts (7 of 7 passed on 12 October 2026 with law 0.1.12); the second the owner-occupier flag; the third the dates past the values the model holds.

Exercise

1. The Vogts' file lacks the agreed rent. Which of these does the model still establish: the income, the rent level, the rent taken into account, the amount?
2. Why is BOTH better for the office than taking the most recent record?
3. A colleague reads "outside the model" on the neighbour's file and writes to her: "You are not entitled to Wohngeld." What is wrong with that letter?

PART IV

What the Model Gives the Authority



Audit and Reproduction

By the end you can identify a saved answer by its fingerprints, follow an annex value back to the gazette, replay the answer, and state what replay establishes and what still requires separate review.

Lawyer: distinguish reproducibility from correct interpretation.

Analyst: preserve the question, the facts and the date.

Developer: compare program, case and result fingerprints, and replay.

1. The question

A year after the decision, an internal audit — or a court — takes up the Vogts’ case. The reviewer asks three things: *Which rules and values produced €485? Were they the ones in force on 1 January 2025? Can we get the same answer again today, from the saved bundle, and know that nothing was changed?*

2. The saved answer

When a question is asked with `--out`, the model writes a folder with the question, the program it ran, the case and the result:

```
law ask <case package> --case Familie2025 --query-json \
    queries/wohngeld-485.json --out runs/familie-2025
```

```
runs/familie-2025/
ask.json      the question and how it was asked
request.json  the program, the case and the question, as
               evaluated
result.json   the answer with its proof graph
bundle.json   the saved bundle for replay: inputs,
               snapshot, sources
index.json    the package index: the canons the case
               resolved, with their hashes
```

3. The fingerprints

The manifest of the answer names, among others:

Fingerprint	What it identifies	Value for the Vogts
program	the rules and parameters evaluated	sha256:8a69e9ad...
case	the facts and the context of the case	sha256:8eb8b743...
legal time	the date of the question	2025-01-01
semantics	the version of the language's rules	law.core/0.2.11

Each value used from an annex carries its own fingerprint too. The node of the formula names the rows it read — WertBMitglieder4#2025-01-01 and its siblings — and for each the hash of the dataset it came from. Each row has a source anchor: the text of Anlagen 1–3 in the second WoGFV. And the canon pins the issue itself: BGBl. 2024 I Nr. 314, taken from recht.bund.de, with the SHA-256 fingerprint of its bytes. So the chain runs from the €485 to the formula node, to the row of Anlage 2, to the annex text, to the gazette issue — each link a hash, not a recollection.

4. Repeat the answer

```
law eval runs/familie-2025
```

`law eval` evaluates the saved answer again — without the source files or a rebuild — and must give the same bytes. On 12 October 2026 the replay of the Vogts’ answer saved in the book’s reproduction kit was **byte for**

byte identical to the saved `result.json`: both files have the SHA-256 fingerprint `e5ce59dd...`. The full fingerprints are listed in Appendix D.

The reviewer's third question is answered: the same program, the same case and the same date give the same answer, and anyone can check that by comparing two fingerprints.

5. Change one thing

Replay the bundle of `Familie20241231` instead. The program fingerprint is the same — one program holds both editions — but the case fingerprint differs (another date: `sha256:cba33940...`), the formula node names the rows `#2023-01-01`, their anchor is the *Wohngeld-Plus-Gesetz*, BGBl. I 2022 S. 2160, and the amount is €463. The fingerprints tell the reviewer *what* differs between two answers before anyone reads a number.

6. What replay does not establish

Replay proves identity, not correctness:

- that the program reproduces its answer — **yes**;
- that the program reads the Act correctly — **no**: that is a legal review of the rules, helped by the scenarios of Chapter 5 and the ministry's examples;
- that the facts were true — **no**: that is the caseworker's file;
- that a later version of `law` gives the same bytes — **not shown in this edition**: the replay above ran on the version that recorded the answer, `law 0.1.12`;
- that the pinned texts match the certified originals — **only as far as recorded**: the canon labels its copies by origin, and a check against the certified gazette is a separate step.

A fingerprint identifies saved bytes; legal interpretation requires a separate review. Keeping the two apart is what makes an audit honest.

7. What this gives the authority

For an internal audit, a court or a *Widerspruch* — the household's objection to the decision — the office can hand over one folder: the question, the facts, the program, the answer and the proof. The reviewer

can replay it on their own machine and get the same bytes, follow every annex value to the gazette, and focus the legal review on the rules and facts — not on reconstructing what the software did on the day.

Whoever drafts the Bescheid takes from the same answer the amount, the legal date with the edition and annex rows applied, and the facts with their origin — the material for the essential factual and legal grounds a written decision must state (§ 35 Abs. 1 SGB X, Chapter 0). The answer does not write the Bescheid: its wording, and everything the model does not compute, stay with the authority.

8. Where Catala stands on this

Catala 1.3.0 provides computation traces for inspecting intermediate steps (Catala book, ch. 2.4⁹), alongside scenario tests and continuous integration (ch. 3.3¹⁰). Arxo builds saved query bundles and program, case and result fingerprints into replay; the Catala 1.3.0 book does not describe this built-in query-bundle form. An application can preserve its inputs and outputs. Here the authority can check a specific computation's identity while leaving legal interpretation open to independent review.

Exercise

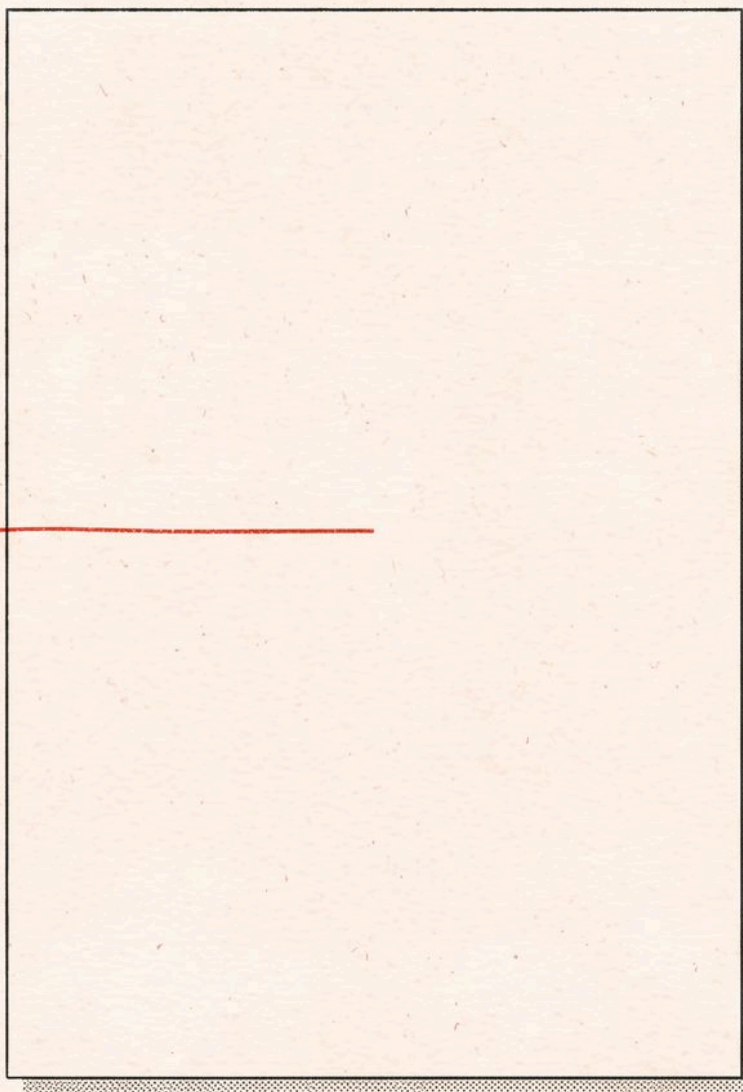
1. Two saved answers for the Vogts have the same program fingerprint but different case fingerprints. What can you conclude before reading the amounts?
2. A replay gives a result with a different fingerprint from the saved one. Name two places to look first.
3. A reviewer says: “The replay is identical, so the decision is lawful.” What is missing from that argument?

⁹<https://book.catala-lang.org/en/2-4-states-dynamic.html>

¹⁰<https://book.catala-lang.org/en/3-3-0-test-ci.html>

PART V

Your Turn



FINAL EXERCISE

Your Own Wohngeld Case and a Country Pilot

By the end you will have built and checked a second household's case and drafted a pilot brief for a benefit in your own country.

Lawyer: which provisions decide each answer, and which the authority keeps.

Analyst: which facts you must request and which change the answer.

Developer: the case, the questions, and the saved answers.

1. The household

Rechenbeispiel 2 of the BMWSB brochure as of 1 January 2025: a single man in Ludwigshafen am Rhein receives unemployment benefit (*Arbeitslosengeld*, ALG I) of €1,350 a month and rents a flat for €470 a month without heating.

The ministry's example gives $Y = €1,350.00$, $M = €580.40$ and a Wohngeld of €88. The canon's 2025 suite reproduces all three.

2. Your submission

Work through five steps. For each, write your prediction first, then the provision, then the answer you obtained.

1. **The rent level.** Which rent level does the WoGV assign to Ludwigshafen am Rhein, and which ceiling plus climate component follows for one member in 2025?

2. **The income.** ALG I is a wage-replacement benefit under § 14 Abs. 2 Nr. 6 WoGG with § 32b Abs. 1 S. 1 Nr. 1 Buchst. a EStG. Why does the example deduct no flat-rate work expenses, and which § 16 deductions apply?
3. **The rent taken into account.** Compute M.
4. **The claim.** What must the case contain, besides the facts above, for the claim to €88 to be *established* rather than *requiring judgment*?
5. **The boundary.** Change one fact so that the model returns each of: *not established*; a ground of exclusion; the flag *outside the model*. Name the fact and the provision each time.

3. Worked solution

1. The WoGV lists Ludwigshafen am Rhein, Stadt at **Mietenstufe IV**. For one member in 2025, Anlage 1 gives a ceiling of €511, and § 12 Abs. 7 a climate component of €19.20: **€530.20** in all.
2. ALG I is not income from employment, so the employee's flat rate of § 9a EStG does not apply; the ministry notes "kein Arbeitnehmer-Pauschbetrag". The example assumes that no § 16 deductions apply, so $Y = \text{€1,350.00}$.
3. The rent of €470 is below €530.20, so all of it counts; plus heating relief for one member (§ 12 Abs. 6), €110.40: $M = \text{€580.40}$.
4. The Wohngeldbehörde's assessment under § 21 Nr. 3, recorded as adjudicated, that claiming would not be abusive. Without it the amount is established and the claim requires judgment (Chapter 6). Beside it, the case must state the § 20 fact — that he is not eligible in principle for training support (bafoeg in the Vogts' case) — and the other premises the Vogts' case already holds; without the § 20 fact the claim does not follow.
5. Examples:
 - *not established*: remove the place of residence — no rent level, no ceiling, no amount (Chapter 8);
 - *exclusion*: enter that he receives SGB II support with housing costs — § 7 Abs. 1 S. 1 Nr. 1 excludes him, § 21 Nr. 2 denies the claim (Chapter 7);
 - *outside the model*: make the application a Lastenzuschuss — the flag names the reason (Chapter 8).

4. What this gives the authority

Two households now run through the same model: a family of four in Weimar and a single man in Ludwigshafen. Different rent levels, different kinds of income, the same sections, the same kinds of answer. That is the point of an executable model for a Wohngeldbehörde: the next file is not a new calculation by hand but a new set of facts.

5. Where Catala stands on this

Catala's founders showed that a statute can be written as code beside its own text, and that doing so finds real discrepancies. This exercise builds on that idea and adds what an office needs around the number: the edition of the date, the authority's open questions, exclusions resting on other decisions, and answers that can be replayed.

6. Country-pilot brief

Draft one page for a benefit in your own country, using the German case as the pattern:

Field	The German case	Your benefit
Benefit and paying authority	Wohngeld; the Wohngeldbehörde of a district or city	
Texts and where they are published	WoGG, WoGV, EStG; BGBl.	
How amounts change	an ordinance every second year (§ 43); the switch on the day (§ 44)	
Last legal date covered by this model	31 December 2026	
Questions the authority keeps	§ 21 Nr. 3 “missbräuchlich”; § 5 “nach verständiger Würdigung”	
Facts decided by other authorities	§ 7: basic income support with housing costs	
Official worked examples	BMWSB, 2023 and 2025: 18 of 21 executed	
What stays outside the first model	Lastenzuschuss; some kinds of income	
Rules on automated decisions	§ 31a SGB X (Wohngeld is social law; § 35a VwVfG only for comparison)	
Reuse of the texts	§ 5 UrhG: statutes are free	

Acceptance checklist

- Every number in the brief comes from an official example or a run of the model, with its date.
- Every question the text leaves to an authority is named, with the authority.
- The last legal date covered by the model is stated, and what the model returns after it.
- What stays outside the model is listed as outside the model, not as “not entitled”.
- The legal basis for reusing the texts is checked.

APPENDIX A

The Answer Card and Its Statuses

Every answer in this book comes from one question asked of the model about one file on one date. The chapters show it as a table row, an answer card or a short excerpt; behind it is a saved answer document. This appendix lists what that answer carries and the statuses the book uses.

What an answer carries

Part	What it says	First shown
The question	the statement or amount asked about, the file and its legal date	Chapter 0
Facts used	the family's data, each with its origin: declared by the household (<code>case_input</code>) or decided by an authority (<code>adjudicated</code>)	Chapter 3
Parameters	the values of the annexes, each tied to the row of the Anlage in force on the date and to the issue of the Bundesgesetzblatt that published it	Chapter 1
Operations	a proof graph, rule by rule, from the facts to the result — for the formula, M, Y, the unrounded <code>z4</code> and the rounded amount	Chapter 4
Sections	the section behind each rule, in the edition in force on the date of the question	Chapter 3
Truth status	whether the facts and rules support the statement, its opposite, neither, or both	Chapter 1
Requests	a question the Act leaves to an authority, with its addressee	Chapter 6
Issues	why there is no amount where there is none: missing values, a conflict of records	Chapters 1, 8
Fingerprints	the program, case and result hashes that let the answer be replayed	Chapter 9

The answer card

Every volume of this series shows a decision on the same card: the question, the legal date, the answer with its status, the claim or right where there is one, the decisive provisions, the sources, the readings applied, the open questions and the fingerprints. The card for the Vogts' monthly Wohngeld on 1 January 2025, as Chapter 3 shows it:

Question	monthly Wohngeld and the claim to it, Familie Vogt
Legal date	1 January 2025 — values of the second WoGFV
Answer	€485 a month (Mietzuschuss) — established
Claim	€485 — established
Decisive provisions	the amount — § 19 Abs. 1–2 with Anlagen 1–3, last step Anlage 3 Nr. 3; the claim — §§ 19–21
Sources	§ 19 Abs. 1, 2 WoGG; Anlagen 1–3 as published in BGBl. 2024 I Nr. 314
Readings applied	intermediate values longer than ten decimal places are rounded half up to ten (Chapter 4)
Open questions	none — the § 21 Nr. 3 assessment is recorded as decided by the authority
Fingerprints	program sha256:8a69e9ad..., case sha256:8eb8b743... (Chapter 9)

Truth status

Status	In this book	Meaning
TRUE_ONLY	<i>established</i>	The facts and rules support the statement and do not support its opposite.
FALSE_ONLY	<i>refuted</i>	They support the opposite and do not support the statement. Not met in this book.
NEITHER	<i>not established</i>	They support neither. This is not zero and not a refusal: often a fact is missing, and the model names the open premise.
BOTH	<i>contradiction</i>	They support both. The answer keeps both, with their sources, for review instead of choosing silently.

Evaluation status and issues

The truth status says what follows; the evaluation status and the issues say whether the model could answer at all, and if not, why.

Status or issue	Meaning	Chapter
REQUIRES_JUDGMENT	The Act leaves a question to the authority — under § 21 Nr. 3, whether claiming would be abusive. The answer carries a request naming the Wohngeldbehörde, the question and the answer the claim needs.	6
MISSING_PARAMETER_VALUE	The model holds no annex value for the date of the question — for 1 January 2027, after the last values this model holds. It does not say that no values are in force.	1, 8
EDITION_NOT_APPLICABLE	The model holds no edition for the date — before 1 January 2023.	1
KEY_CONFLICT	The file holds two values where the model admits one, such as two agreed rents for one tenancy.	8

A flag, not a status

Where a case lies outside what the model covers — an owner-occupier's *Lastenzuschuss*, some co-residents whose share of the rent § 11 Abs. 2 sets aside, a residence title under § 3 Abs. 5 Satz 3 — the model establishes the flag `ausserhalb_des_modells` and gives no amount. The flag is a statement about the model, not about the person's entitlement (Chapter 8).

APPENDIX B

Glossary: German → English

German terms as the chapters gloss them where they first appear.

German	English	Chapter
<i>Anlage</i>	annex	0
<i>Bescheid</i>	decision notice	0
<i>Bewilligungszeitraum</i>	grant period	1
<i>Bruttokaltmiete</i>	gross rent without heating	0
<i>Bundesgesetzblatt (BGBl)</i>	federal law gazette	0
<i>Fortschreibung</i>	biennial update	0
<i>Gesamteinkommen</i>	assessed household income	0
<i>Grundsicherungsgeld</i>	basic income support under SGB II, so named since 1 July 2026 (before: <i>Bürgergeld</i>)	7
<i>Haushaltsmitglied</i>	household member	0
<i>Höchstbetrag</i>	rent ceiling	2
<i>Klimakomponente</i>	climate component	2
<i>Lebensmittelpunkt</i>	centre of one's life	2
<i>Mietenstufe</i>	rent level I–VII	0
<i>Mietzuschuss / Lastenzuschuss</i>	rent subsidy / owner-occupier subsidy	0
<i>missbräuchlich</i>	abusive	6
<i>Rechenbeispiel</i>	worked example	0
<i>Widerspruch</i>	objection to a decision	9
<i>Wohngeld</i>	housing benefit	0

German	English	Chapter
<i>Wohngeld-Plus-Gesetz</i>	amending Act of 2022 (Anlagen 2 and 3 for 2023– 2024)	1
<i>Wohngeldbehörde</i>	housing-benefit authority	0
<i>Wohngeldfortschreibungsverordnung</i> (WoGFV)	ordinance that updates the annex values every second year	1
<i>zu berücksichtigende Miete</i>	rent taken into account	0

Index of Sections and Publications

Sections and publications cited in the chapters, with the chapters that cite them; a section cited without the name of its Act belongs to the WoGG. German statutes and ordinances are quoted from the consolidated texts on gesetze-im-internet.de, provided by the Federal Ministry of Justice, and the annexes from the issues of the Bundesgesetzblatt named below; as statutes and official decrees they are not protected by copyright (§ 5 UrhG). Each chapter names the edition it quotes and the date it applies.

Wohngeldgesetz — Housing Benefit Act (WoGG)

Section	Chapters
§ 3	7–8
§ 5	0, 2–3, 6, 8, final exercise
§ 6	3
§ 7	2–3, 6–8, final exercise
§ 8	6–7
§ 9	2–3
§ 10	8
§ 11	0, 2–3, 7–8
§ 12	0–3, final exercise
§ 13	0, 2–3
§ 14	0, 2–3, final exercise
§ 15	0, 2

Section	Chapters
§ 16	0, 2–4, final exercise
§ 17	1–2, 8
§ 18	2
§ 19	0, 2–4, 6
§ 20	0, 3, 6, final exercise
§ 21	0, 3, 5–7, final exercise
§ 41	1
§ 43	0–1, 4, final exercise
§ 44	0–1, final exercise

Wohngeldverordnung – Housing Benefit Ordinance (WoGV)

Section	Chapters
§ 1	3
§ 6	2

Einkommensteuergesetz – Income Tax Act (EStG)

Section	Chapters
§ 2	2
§ 9a	2–3, final exercise
§ 32b	final exercise

Sozialgesetzbuch Zehntes Buch – Social Code, Book Ten (SGB X)

Section	Chapters
§ 31a	0, 6, final exercise
§ 35	0, 9
§ 104	7

Verwaltungsverfahrensgesetz – Administrative Procedure Act (VwVfG)

Section	Chapters
§ 35a	6, final exercise

Urheberrechtsgesetz – Copyright Act (UrhG)

Section	Chapters
§ 5	final exercise

Gazette issues and parliamentary papers

Publication	Chapters
BGBI. 2024 I Nr. 314	1, 3, 9
BGBI. 2026 I Nr. 107	7
BGBI. I 2021 S. 1369	1
BGBI. I 2022 S. 2160	1, 9
BT-Drs. 21/8284	1

Reproducing the Examples

Every answer in this book was produced by the `law` command-line tool, either from a *case package* — a small package that holds fictional files, each with its facts and legal date, and imports the canons whose rules apply — or from the declared scenarios of the Wohngeld canon itself. The rules are never copied into the case.

What you need

- The `law` command-line tool, version 0.2.2 or later (`npm i -g @arxo/cli`), executing `law.core/0.2.11`. The chapters say which version recorded each table: on 12 October 2026, `law 0.1.12` in a checkout of the Arxo repository. The reproduction kit below was checked the same day with the released `law 0.2.2`: the same amounts and statuses.
- The reproduction kit of this book: two public repositories.
 - the canons, `github.com/arxohq/arxo-wohngeld`: the three German canons, each with its sources, pinned legal texts and scenarios, and a registry from which the case packages install them;
 - the examples, `github.com/arxohq/law-as-code-germany-examples`: the case packages and the saved answers of Chapter 9.

Getting the kit

```
r=https://github.com/arxohq
git clone $r/arxo-wohngeld.git
git clone $r/law-as-code-germany-examples.git examples
```

The case package of the chapters' commands, `<case package>`, is `examples/familie-weimar`. The canon, `<canon>`, is the folder `arxo-wohngeld/packages/de.social.wohngeld`: its dependencies are pinned inside it, so there is nothing to install.

`law install` never picks versions: it restores exactly what `law.lock` pins and checks every hash. In a checkout of the Arxo repository the same package and canon are `corpus/laws/de/wohngeld/examples/familie-weimar` and `corpus/laws/de/wohngeld`.

The case package

All the files of the Vogts and of the excluded woman are in one case package, `familie-weimar` — the `<case package>` of the chapters' commands. It holds four files and six saved questions.

File	Who and when	Chapters
Familie2025	the Vogts on 1 January 2025, the § 21 Nr. 3 assessment recorded as decided	0–4, 6, 9
Familie20241231	the same family on 31 December 2024	0, 1, 9
FamilieOhneWuerdigung	the Vogts on 1 January 2025 without the authority's assessment	3, 6
AlleinGrundsicherung	a woman in Weimar receiving SGB II support with housing costs, on 1 March 2025	7

The questions are JSON files in `queries/`:

Question file	Asks whether
einkommen-1710.json	the monthly assessed income is €1,710.00
miete-777-80.json	the rent taken into account is €777.80
wohngeld-485.json	the monthly Wohngeld is €485
wohngeld-463.json	the monthly Wohngeld is €463
anspruch-485.json	a claim to €485 exists
kein-anspruch-alle-ausgeschlossen.json	no claim exists because all household members are excluded

One fact changed

The variations of Chapters 2, 3, 4 and 8 — the Vogts’ case with one fact changed — are five cases of a second case package of the kit, `familie-weimar-varianten`. Its questions collect the value (`collect-y`, `collect-m`, `collect-w`, `collect-n` for Y, M, the Wohngeld and the number of members).

File	Change against Familie2025	Answer	Chapters
VMiete850	rent €850	M €998.20, €644	2, 3, 4
VSteuer	income tax also expected	Y €1,496.25, €542	2, 3, 4
VWerbung2000	work expenses €2,000	Y €1,658.6666666667 (ten decimal places, rounded half up), €499	2, 3
VOhneMittelpunkt	Markus’s life centre not established	3 members, €351	2, 3, 8
V20270101	legal date 1 January 2027	MISSING_PARAMETER_VALUE	8

The canon’s scenarios

The other answers are scenarios of the Wohngeld canon — the `<canon>` of the chapters’ commands:

Scenario file	What it holds	Chapters
tests/bmwsb-2023.lawtest, tests/bmwsb-2025.lawtest	the ministry's worked examples, 9 and 9 tenant examples executed	5, final exercise
tests/redaktionen.lawtest	the change of edition and the edges of the values the model holds	1, 8
tests/tatsachen.lawtest	missing and conflicting facts	8
tests/ausschluss-und-urteil.lawtest	exclusions under § 7, the authority's judgment, the owner-occupier flag	6, 7, 8

Asking a question

Work in the case package directory — from the folder where you cloned the kit:

```
cd examples/familie-weimar
law install --registry arxo=../../arxo-wohngeld/registry
law ask . --case Familie2025 \
  --query-json queries/wohngeld-485.json --out \
    "$ (mktemp -d)"
```

`--case` names the file, `--query-json` the question; `--out` saves the full answer — status, proof and fingerprints — in a directory.

Running the scenarios

In the canon folder, `arxo-wohngeld/packages/de.social.wohngeld`:

```
law test . --file tests/bmwsb-2025.lawtest
```

Each line reports a scenario as passed or failed, and a total. On 12 October 2026 the 2023 suite passed 9 of 9, the 2025 suite 10 of 10, the edition suite 6 of 6, the suite of missing and conflicting facts 7 of 7, and the suite of exclusions and judgments 16 of 16; in `arxo-wohngeld`, with the released `law` 0.2.2, all 87 scenarios of the canon passed.

Replaying a saved answer

The kit keeps the two answers of Chapter 9 in `familie-weimar/runs/`. Replay one in a copy:

```
d="$(mktemp -d)"
cp -R runs/familie-2025 "$d"
law eval "$d/familie-2025"
shasum -a 256 runs/familie-2025/result.json \
    "$d/familie-2025/result.json"
```

`law eval` evaluates the saved answer again — without the source files or a rebuild — and must give the same bytes. Compare the program and case fingerprints in the manifest and the fingerprint of `result.json` in the original and the replay: if they match, the same program gave the same answer to the same case. That shows repeatability, not legal correctness — Chapter 9 explains the difference.

The saved answers and their fingerprints, recorded on 12 October 2026; each replay was byte for byte identical:

Fingerprint	Value
program, both answers	sha256:8a69e9ad8320d428a783c2a2e40bb0de53eb469ff99427bad732eaa21d0b749c
case Familie2025 (€485)	sha256:8eb8b74380524355eada4c2c6b4811dc2e7a1fe166ce166dd94283d27ccf3f22
case Familie20241231 (€463)	sha256:cba339409be3cd65a5b4701157e4a8331a7a265b2fa02a6e0a33c0b165e59bbc
result.json, Familie2025	e5ce59dd0f27c666410c9d247fad47b9ed75a939b052e360cb64fb6fe6790eb4
result.json, Familie20241231	f685d323d60ca09fbadd2be9940e09a7855fa3b0b4e593b0bf90c25d8b3f1735
semantics	law.core/0.2.11

If your answer differs

Check, in this order: the version of `law`; the legal date and the facts of the file; the editions and annex rows the canon pins. A later edition of a text, or a corrected fact, gives a new answer to a new question — not a failed reproduction of the old one.

Solutions to the Exercises

Chapter 0 — What a Wohngeldbehörde Decides, and Why It Is a Law-as-Code Problem

1. *Not enough facts* — the amount is *not established*. Without the rent, the formula has no M , so there is no amount to compute; returning €0 would state that the family is owed nothing, which nobody has decided.
2. The monthly Wohngeld depends on the date (€463 on 31 December 2024, €485 on 1 January 2025). In this case the income (€1,710.00) and the rent taken into account (€777.80) are the same on both dates; Chapter 2 shows why.
3. The claim needs more than the formula: the amount must be at least €10 (§ 21 Nr. 1), not all members may be excluded (§ 21 Nr. 2), and claiming must not be abusive — an assessment the model leaves to the Wohngeldbehörde (§ 21 Nr. 3). Chapter 6 shows what happens when that assessment is still missing.

Chapter 1 — Which Text Applies? The Bundesgesetzblatt and the § 44 Transition

1. It must decide again, of its own motion, for 1 January to 30 September 2025, applying the values of the second WoGFV: €463 a month until 31 December 2024 and €485 a month from 1 January 2025.
2. It returns no amount, with the issue `MISSING_PARAMETER_VALUE`. A carried-over value would look like a lawful amount without saying

which text it rests on; the refusal names what is missing — the values for the new period.

3. In Munich the rent (€1,225) is above the ceiling, so the ceiling decides the rent taken into account, and the ceiling rose. In Weimar the rent (€580) stays under the ceiling on both days, so the rent itself is used: $€580 + €197.80 = €777.80$ in both editions.

Chapter 2 — Household, Rent and Income

1. $€26,880 - €2,000 = €24,880$; minus 20 % = €19,904 a year; $Y = €19,904 \div 12 = \mathbf{€1,658.6666666667}$ — the model holds Y at ten decimal places (rounded half up), not rounded to the cent — and the Wohngeld becomes €499. Proven expenses replace the flat rate only when they are higher, as here.
2. The rent (€850) is now above the ceiling plus the climate component (€800.40), so the ceiling counts: $M = €800.40 + €197.80 = \mathbf{€998.20}$, and the Wohngeld becomes €644.
3. Because § 5 Abs. 1 makes membership depend on facts — the centre of one's life, the relationship to a member — not on registration. The case must establish those facts: a person whose life centre in the flat is not established is not counted (section 6), and a person whose status under § 7 is unknown leaves the household *not established* (Chapter 8).

Chapter 3 — One Family, One Decision: Weimar, Mietenstufe III, €485

1. The place of residence — wohnort — because the rent level is read from the WoGV's list for the municipality (§ 12 Abs. 1 WoGG with the Anlage to § 1 Abs. 3 WoGV). The case has no field for the level; it follows from the place.
2. § 11 Abs. 1 WoGG: the rent counts only up to the ceiling of Anlage 1 (§ 12 Abs. 1) plus the climate component (§ 12 Abs. 7) — €800.40 for four members at level III in 2025.
3. The assertion *behoerde-21-nr3* with origin adjudicated: the Wohngeldbehörde has assessed that claiming would not be abusive. Without it, the claim would require judgment.

Chapter 4 — The § 19 Formula: Anlagen 2 and 3 and One Rounding

1. With the values of 31 December 2024:

$$\begin{aligned} z1 &= 0.01 + 0.0002251 \cdot 777.80 + 0.00002 \cdot 1,710 \\ &= 0.01 + 0.17508278 + 0.0342 \\ &= 0.2192827800 \end{aligned}$$

The remaining steps give $z2 = 374.9735538000$, $z3 = 402.8264462000$, $z4 = 463.2504131300$, and €463.

2. Because Anlage 3 Nr. 2 prescribes ten decimal places for the intermediate steps and Nr. 3 allows rounding to the euro only at the end. Rounding earlier is a different calculation and can change the amount by a euro.
3. When $z4$ lies very close to a half euro, so that the small differences between rounding and cutting off at the tenth decimal place — carried through $z2$ and $z4$ — put it on different sides of the 50-cent line.

Chapter 5 — How a Scenario Is Written

1. An expectation copied from the engine only checks that the engine agrees with itself. The PDF is an independent statement of the answer; agreement with it is evidence.
2. Two errors that cancel out — for example a wrong ceiling and a wrong heating relief that happen to give the right total — or the right total reached through the wrong path.
3. No. They are owner-occupier subsidies (Lastenzuschuss), which the model declares outside its scope; for them it returns a flag, not an amount. The scope line — 18 of 21 executed — says so openly.

Chapter 6 — Sum Known, Claim Pending: § 21 Nr. 3 and the Wohngeldbehörde

1. Only the claim. The income, the rent and the amount stay established; the claim to €485 becomes *not established*, and “no claim, ground § 21 Nr. 3” becomes *established*.

2. Because the model leaves that assessment to the Wohngeldbehörde. Treating silence as “not abusive” would turn the office’s decision into a default of the software; the household would receive a claim nobody competent has confirmed.
3. A threshold. § 21 Nr. 3 names “considerable assets” only as an example and sets no amount; a fixed limit would replace the office’s assessment with a number the Act does not contain. If an authority adopts internal guidance with a threshold, that guidance is a separate source and must be named as such.

Chapter 7 — Excluded by Another Authority’s Decision: § 7 WoGG

1. The exclusion no longer applies (§ 7 Abs. 1 S. 3 Nr. 1): she is a counted member, and the model computes Wohngeld for her household, provided the other facts are complete.
2. Because § 7 Abs. 1 excludes recipients only “wenn bei deren Berechnung Kosten der Unterkunft berücksichtigt worden sind”. Receiving the benefit without housing costs does not exclude.
3. Because § 7 excludes person by person: the son receives SGB II support with housing costs, the father does not. So not all members are excluded (§ 21 Nr. 2 does not apply), and the household’s Wohngeld is computed for the one counted member, with the rent taken in proportion under § 11 Abs. 3. (Had the excluded son been the applicant, § 3 Abs. 4 would have kept him entitled to claim for the household.)

Chapter 8 — Missing Facts, Conflicting Records and the Model’s Boundary

1. The income (€1,710.00) and the rent level (Mietenstufe III, from the place). Not the rent taken into account, and so not the amount.
2. Because the most recent record is not necessarily the correct one. BOTH keeps both records and their sources and leaves the resolution to the person who can check them; a silent choice would hide the conflict in the decision.
3. “Outside the model” says the model does not cover owner-occupier subsidies; it says nothing about her entitlement. She may well be

entitled to a Lastenzuschuss. The office must decide her case by other means.

Chapter 9 — Audit and Reproduction

1. The same rules and parameters were evaluated, on different facts or a different date. Here: `Familie2025` and `Familie20241231` differ only in the date, which selects other rows of the annexes.
2. The program (was the bundle replayed with a different program or language version — compare the `semantics` and program fingerprints in the manifest) and the inputs (was the saved bundle changed — compare the case fingerprint).
3. Replay shows that the computation is reproducible. It does not show that the rules read the Act correctly or that the facts were true; those need a legal review and the file.

Where Catala Stands

Most chapters close with a short section on how Catala — and, in Chapter 7, GETTSIM — meets the same need. Those sections describe the other tools as their own documentation presents them. They were checked against Catala 1.3.0 (released 1 October 2026) and the Catala book on 11 October 2026; the date is printed here once instead of under every section.

Tools change. If something has changed, tell us at hi@arxo.io. Corrections are published with the next edition and listed under Corrections.

Chapter	What the chapter compares
1	dates and pinned publications
3	exact arithmetic of the formula
4	rounding and intermediate values
5	tests and expected statuses
6	a question left to the authority
7	exceptions and inputs decided elsewhere
8	missing facts and contradictory records
9	traces, fingerprints and replay
Final exercise	statute as code — the idea this book builds on

Acknowledgements

This book stands on German work that is older than the phrase “law as code”. Gottfried Wilhelm Leibniz, a doctor of law before he was known as a mathematician, argued in 1667 in his *Nova methodus discendae docendaeque jurisprudentiae* that jurisprudence could be taught as a method. The Bürgerliches Gesetzbuch, in force since 1 January 1900, gave the world a general part and a systematic order that many codes still follow. Wilhelm Steinmüller’s *EDV und Recht* (1970) and Herbert Fiedler’s work in Bonn helped open the field of *Rechtsinformatik*; Hesse passed the world’s first data-protection act the same year. Since 1985 juris has kept German law searchable; gesetze-im-internet.de made federal law free to read, and DigitalService is building NeuRIS, a new public portal for legal information.

Today’s work continues that line. GETTSIM models German taxes and transfers in the open. The Digitalcheck asks every federal bill whether it can be executed digitally. And Catala — the pioneer of statute-as-code from Inria — showed that a statute can be written as code beside its own text.

This book follows an independent Arxo model built from published texts. Where it compares tools, it names the version and the date of the check (Appendix F).

Sources

- **Wohngeldgesetz (WoGG) and Wohngeldverordnung (WoGV)** — consolidated texts from gesetze-im-internet.de, provided by the Federal Ministry of Justice; German statutes and ordinances are free of copyright under § 5 UrhG.

- **Bundesgesetzblatt** — BGBl. 2024 I Nr. 314 (second WoGFV) and BGBl. 2026 I Nr. 107 from recht.bund.de; BGBl. I 2021 S. 1369 and BGBl. I 2022 S. 2160 from the copies published by OffeneGesetze.de.
- **Einkommensteuergesetz (EStG)** — § 2 and § 9a, for the definition of income the WoGG refers to.
- **Federal Ministry for Housing, Urban Development and Building (BMWSB)** — *Rechenbeispiele 2023* and *Beispiele für die Berechnung des Wohngelds*, as of 1 January 2025. Their worked examples are the expectations of the model's scenarios; the book reuses their facts and numbers, not their text.
- **German Bundestag** — BT-Drs. 21/8284 of 30 September 2026, the government bill for 2027, cited for its status only.

If something in this book is wrong or out of date, tell us at hi@arxo.io and we will correct it.

Corrections

Corrections to this edition are listed here, with the chapter, the change and the date, and published with each new version of the book.

None yet.

To report an error — in the law, the numbers, the German or the English — write to hi@arxo.io. Please name the chapter and, if you can, the section or the table.